Optimal Information Acquisition and Sharing Decisions: Joint Reviews on Crowdsourcing Product Design

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ABSTRACT

The acquisition and sharing of reviews have significant ramifications for the selection of crowdsourcing designs before mass production. This article studies the optimal decision of a brand enterprise regarding the acquisition/sharing of crowdsourcing design reviews in a supply chain. The authors consider an analytical model where the brand enterprise can privately acquire the manufacturer's review (MR) of crowdsourcing product designs and choose one of two information-sharing schemes—optional or mandatory sharing—to disclose MR to the key opinion leaders (KOLs), which help them to produce fans' reviews (FR). MR and FR integrate into the joint reviews (JR) that impact prospective consumers' purchase intention. The authors find that mandatory sharing significantly harms the brand enterprise's motivation to obtain MR, yet optional sharing is conducive to boosting JR on crowdsourcing designs. In addition, JR has a ceiling value, implying that excessively high FR and MR could not always enhance the effect of JR on crowdsourcing designs.

KEYWORDS

Crowdsourcing design, information acquisition and sharing, joint reviews, KOLs' reviews

INTRODUCTION

Consumers prefer to tailor their products instead of passively adopting a firm's offerings (Namin et al., 2023). Therefore, constantly launching new customized products is critical for enterprises to meet different individuals' demands (Ogink & Dong, 2019). Traditionally, enterprises rely on their internal R&D to enhance innovation capabilities and yet shoulder heavy financial burdens. However, mobile Internet and smart devices allow enterprises to obtain cost-saving innovation beyond the organization's boundary (Chan et al., 2021). One such innovation paradigm is crowdsourcing, through which enterprises seek solutions that appeal to potential users or prospective consumers (Liu et al., 2020). The fundamental goal of crowdsourcing is to solicit innovative ideas by exploiting

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the collective expertise and wisdom of heterogeneous outside participants in a quick response to customized demands, rather than depending on a limited quantity of internal specified professionals (Liu et al., 2022).

Crowdsourcing has been implemented in both business-to-consumer (B2C) and business-to-business (B2B) markets (Gao et al., 2022). It helps firms effectively embed potential consumers' ideas into their innovation tasks. For example, with crowdsourcing innovation, Xiaomi, as one of the worldwide leading electronics manufacturers, earned over 5.6 million US dollars in 2018 from its smart products (Li et al., 2019), while Dell also has collected more than 16,000 novel ideas from its customers to sharpen its competitive edge via crowdsourcing innovation since 2010 (Bayus, 2013).

Although crowdsourcing allows firms to meet consumers' customized needs, it is still challenging to accurately and holistically assess the quality of crowdsourcing designs and ideas due to its online innovation (Li et al., 2021; Pan et al., 2022). To overcome this obstacle, firms encourage fans on crowdsourcing platform to post their reviews (FR) because a considerable number of consumers utilize online reviews from fans or early consumers to make purchase decisions. Meanwhile, the "herd effect" of online comments psychologically boosts consumers' purchase intentions and weakens their wait-and-see status (Chan et al., 2021).

Typically, consumer reviews are written online after products are purchased or consumed, and they are widely utilized to promote sales in e-commerce, but this kind of consumer review belongs to the after-purchase reviews, unsuitable for before-production crowdsourcing designs (Chen_et al., 2020). In contrast, fan reviews in the crowdsourcing setting, submitted before manufacturing crowdsourcing products, aim to assess crowdsourcing design quality, which means FR is an ex-ante online review. Without FR, if the crowdsourcing designs are turned into finished products, they may not precisely match consumers' customized requirements, leading to consumers' reluctance to purchase, thus such crowdsourcing is a failure. To avoid such potential risks, some firms, including Xiaomi, have employed the ex-ante FR rather than post-purchase reviews to identify the quality of crowdsourcing designs (Li et al., 2021).

Generally, desirable crowdsourcing design quality would include input by all related members because each member (for example in the supply chain) plays a unique role in reappraising the crowdsourcing designs' values from various lenses. Specifically, crowdsourcers are in charge of hosting the crowdsourcing activities, and crowdsourcees contribute to creative solutions based on personalized requirements (Piezunka & Dahlander, 2019). Enthusiastic fans assess the proposed crowdsourcing design quality from the perspective of both ease of use and usefulness (Li et al., 2019). Manufacturers produce and comment from the perspective of production cost, manufacturability, ease of making, and recyclability. In this way, crowdsourcers enable a holistic and precise understanding of crowdsourcing quality. Afterward, the most desirable solutions are selected and put into production (Wong et al., 2021); some firms, such as Xiaomi and Suning, capitalize on the ex-ante FR in conjunction with the manufacturer's review (MR) to judge the crowdsourcing designs before mass production (Li et al., 2021).

In a setting with MR and FR information, crowdsourcers have two information-sharing schemes. One is the optional sharing scheme, wherein crowdsourcers determine whether to share MR with fans. The other is the mandatory sharing scheme, wherein crowdsourcers are mandated to share MR with fans and prospective consumers because, for some special scenarios, the outcomes of information acquisition must be shared with the public. An example of the sharing scheme is seen with Amazon's adoption of blockchain technology to guarantee that some crowdsourcing design information, from designer to supplier to end-users, is visible and reliable across the whole supply chain (Kadadha et al., 2021). Once the corresponding members engage in these blockchain-based supply chain systems, the quality information acquired from the providers can be traceable and publicly observable (Wu et al., 2021). Thus, whether the evaluations are favorable or not, the crowdsourcers cannot hide this information.

Although both sharing schemes are popular in real business, few studies have precisely examined the interplay of FR and MR and the impact of the joint reviews (JR) on a firm's optimal strategies with respect to the crowdsourcing design information acquisition and sharing.

To clearly articulate the relationship between the three types of reviews – MR, FR, and JR – we first explain MR, which is posted by the manufacturer and focuses on the production feasibility, manufacturability, ease of making, and recyclability for crowdsourcing designs. FR is submitted by fans, highlighting crowdsourcing designs in terms of ease of use and usefulness or the level of deviation from their expectations (with respect to categories such as function, shape, color, and style); it is an ex-ante review rather than an after-purchase review (consumer review). Meanwhile, FR is also influenced by MR when the crowdsourcer (enterprise) acquires MR and then shares it with fans. JR combines both MR and FR, thus yielding a metric to holistically assess the crowdsourcing design quality; JR impacts prospective consumers' purchase intentions and further influences the firm's final decision regarding whether or not to put crowdsourcing designs into production.

Spurred by this practice, this paper aims to answer the following three questions:

- (1) Under what conditions do JR on crowdsourcing designs occur in the setting of information acquisition/sharing and subsequently impact the crowdsourcer's (i.e., enterprise's) selection of crowdsourcing designs?
- (2) How does the optimal JR information acquisition/sharing scheme change with the behavior of fans and the brand enterprise?
- (3) How do the crowdsourcer's profits and consumer surplus react to JR under different acquisition/sharing schemes?

LITERATURE REVIEW

This paper investigates the joint reviews on the selection of crowdsourcing design, with the consideration of information acquisition and sharing. The related literature can be categorized into two streams: crowdsourcing innovation and information acquisition/sharing.

Crowdsourcing Innovation

Most extant studies examine how to efficiently promote crowdsourcing innovation activities from three main facets: crowdsourcing motivation (Li et al., 2020a; Wei & Wei, 2020), crowdsourcing organizing (Kadadha et al., 2021; Deng et al., 2019), and crowdsourcing feedback (Hu & Wang, 2021).

Our research is closely related to crowdsourcing feedback because the process of crowdsourcing feedback contains two parts: where to acquire and how to share. The existing literature has addressed three types of crowdsourcing feedback: user, peer, and firm. First, user feedback is responsive comments from consumers. Ogink and Dong (2019) examine how users' feedback may stimulate a focal user's contribution to crowdsourcing communities and identify their joint impact on their next contribution. Wang et al. (2019) divide user feedback into two types (explicit and implicit) and explore the related benefits in software crowdsourcing. Liu et al. (2020) examine the user feedback valence to crowdsourcing idea implementation, including the negative effect of positive feedback, and the positive effect of negative feedback.

Second, peer feedback refers to feedback from other crowdsourcing solvers. Namin et al. (2023) investigate the effect of peer vs. expert feedback valence (negative vs. positive) on the crowdsouring motivation, showing that negative feedback from peers has a negative effect on the likelihood of innovation, but negative feedback coming from experts will motivate individuals to generate better ideas. Third, firm feedback refers to comments from the crowdsourcing host. Liao et al. (2021) explore the impact of firm (i.e., host) and peer feedback on crowdsourcing contributions, showing that firm

and peer feedback has a positive relationship with the quality and quantity of contributions, while social learning positively impacts the quantity but negatively affects the quality. Similarly, Chan et al. (2021) find that the effects of negative firm feedback on the next contribution quality become weak, while the effect of positive peer feedback on the quality of the following contribution is improved.

However, whether the feedback belongs to user, peer, or firm, existing literature studies the crowdsourcing feedback process as a whole and does not separate it into sub-processes (where to acquire and how to share), nor does it examine the interaction between them. The reason for separating the feedback process into sub-processes is that, in the actual crowdsourcing innovation, fans cannot directly obtain the manufacturer's evaluations of crowdsourcing design, thus the enterprise needs to actively acquire them from its cooperating manufacturer. Meanwhile, the manufacturer's evaluations obtained by the enterprise may not be shared with fans; for example, some of the manufacturer's evaluations are only used for improvement of initial crowdsourcing designs and are not revealed to fans. Although Li et al. (2021) also study the joint reviews on crowdsourcing design, there is an implicit assumption in their study that the two sub processes are considered as a whole, which is not in line with reality. Therefore, our work will relax this constraint to explore the impact of joint reviews on crowdsourcing design under information acquisition and sharing.

Information Acquisition and Sharing

Information acquisition and sharing are crucial for supply chain members. It is common for high-tech firms to enhance product innovation by obtaining and sharing information among various agents (Li et al., 2020). A body of related literature focuses on information obtaining/sharing from the competitive or co-petitive perspective.

From a competitive standpoint, Cheong and Kim (2004) examine the effect of competition on firms' disclosing quality to consumers with the consideration of the information sharing cost. Jansen (2008) studies oligopolists' incentives under competition to obtain and share information and finds that duopolists prefer sharing if their goods are sufficiently distinctive. Guo and Zhao (2009) investigate how competition may impact duopoly companies' motivation to reveal quality news, demonstrating that companies disclose less in monopolistic settings. Li and Peeters (2017) study competitors' incentives to acquire and share quality information about their rivals' products as well as the timing of revealing. Markopoulos and Hosanagar (2018) analyze a competitive setting where both production quality and quality sharing are impacted by third parties.

Some scholars have addressed this issue from the co-petitive perspective. Guo and Iyer (2010) consider the downstream sharing scenario where the manufacturers can sequentially acquire consumer information and then influence downstream firms' behavior. Gao et al. (2014) explore the upstream sharing scenario, where a strategic information disclosure/sharing problem is discussed. Guan et al. (2020) evaluate a supplier's information-sharing strategy of directly disclosing with end users and find that the setup of a direct selling outlet can expand market shares, inducing the supplier to implement a disclosure strategy.

Although our paper also concentrates on the acquisition and sharing strategies in the co-petitive setting, there are few studies considering the interplay between feedback information acquisitions from the manufacturer and sharing to fans, which generates JR to impact the crowdsourcing solution selection. In addition, this research involves two sources: the manufacturer offering MR and fans providing FR.

Overall, the main contributions of this paper are threefold.

(1) Although prior literature examines the impact of ex-post users' and peers' feedback on crowdsourcing innovation, the ex-ante fans' and manufacturers' feedback on crowdsourcing design quality is rarely studied in the setting of information acquisition/sharing.

- (2) Extant work takes the feedback process as a whole; this paper separates the crowdsourcing feedback into two sub-processes, where to acquire and how to share, and further examines the interaction between them.
- (3) Unlike prior work (Li et al., 2021) simplifying one case of the joint reviews on crowdsourcing solution selection, we consider four cases: (1) the enterprise does not acquire MR and selects the optional sharing scheme, (2) the enterprise does not acquire MR and selects the mandatory sharing scheme, (3) the enterprise acquires MR and selects the optional sharing scheme, and (4) the enterprise acquires MR and selects the mandatory sharing scheme.

THE MODEL

We consider one enterprise (i.e., crowdsourcer) and one manufacturer, and the enterprise solicits product designs via crowdsourcing in response to consumers' tailored demands. Before the enterprise decides whether to outsource the manufacturer to produce the proposed crowdsourcing designs, the proposals will be evaluated by the manufacturer and fans with the consideration of two information sharing schemes. Noting that key opinion leaders (KOLs) represent most online fans, we use KOLs' reviews to measure FR. Table 1 shows each member and their different roles in accessing the crowdsourcing designs.

Based on Huang et al. (2020), assume that the brand enterprise's crowdsourcing product price is $p=-\xi Q+\vartheta$, where Q represents the crowdsourcing product demand, $\xi>0$ denotes the sensitivity parameter, and ϑ indicates consumers' recognition (reservation price), similar to the work of Bigerna et al. (2019), we also assume that ϑ is influenced by two factors: basic recognition level a_0 and joint reviews (JR) r_J ; therefore, $\vartheta=\left(1+a_0\right)r_J$.

If crowdsourcing design quality reaches the expected profit threshold via JR, then those designs can be transferred into production, and the crowdsourcing design quality reflects two aspects: the manufacturing sector (including the production feasibility, manufacturability, ease of making, and recyclability for crowdsourcing designs) and the usage sector (including the ease of use and usefulness, and among others). Before putting crowdsourcing designs into production, the enterprise first decides whether it needs to acquire the ease-of-manufacturing information of crowdsourcing designs from the manufacturer (MR). Specifically, if MR is acquired, the brand enterprise's information acquisition scheme is expressed as $s_A = a$; if MR is not acquired, $s_A = na$, where $s_A \in \{a, na\}$. After the

Members	Define and explanation	Roles	
Brand enterprise	Crowdsourcing host or crowdsourcer; manufacturing outsourcer	In charge of hosting the crowdsourcing assessment activities.	
Manufacturer	Manufacturing for the brand enterprise; one type of reviewers	Assessing the proposed crowdsourcing design quality from the manufacturing perspective.	
Fans	Another type of reviewers	Assessing the proposed crowdsourcing design quality from the perspective of both ease of use and usefulness. The typical most fans' reviews.	
Key opinion leaders (KOLs)	Representative of most fans' reviews		
Consumers	Those who intend to purchase crowdsourcing product after crowdsourcing designs being selected and produced	Being influenced by assessments from manufacturer and fans (KOLs) or JR.	

brand enterprise obtains MR, then it decides whether to share it with KOLs. If it is shared with KOLs, the brand enterprise's information sharing scheme is denoted by $s_{\rm g}=s$; otherwise, $s_{\rm g}=ns$.

Assume that MR (r_m) and the crowdsourcing design quality level (q) are uniformly distributed over $\left(0,\overline{r}_m\right]$ and $\left(0,q_{max}\right)$; if the manufacturer truly and credibly evaluates the qualities of crowdsourcing designs, it indicates that q_{max} equals \overline{r}_m . Conversely, if the brand enterprise is unwilling to acquire MR ($s_A=na$), it cannot precisely know the manufacturer's reviews r_m ; instead, the enterprise and KOLs merely rely on its prediction or the crowdsourcing product design quality distribution, which is uniformly distributed over $\left(0,\overline{r}_m\right)$ to infer the crowdsourcing product recognition.

In addition, after acquiring MR, the brand enterprise will decide its sharing scheme $s_S \in \{s, ns\}$, that is, share MR with KOLs ($s_S = s$) or not ($s_S = ns$), thus helping KOLs to assess the crowdsourcing designs (i.e., FR is influenced by MR). In this regard, the enterprise and KOLs can better understand the different members' reviews on crowdsourcing designs; this is the reason some firms like Xiaomi introduce MR to share with KOLs (Li et al., 2021; Li et al., 2019).

Note that whether the brand enterprise reveals MR or not, KOLs invariably post their reviews online, because KOLs evaluate the crowdsourcing product design out of their enthusiasm, and their corresponding costs are negligible. Without loss of generality, we normalize FR information acquisition cost to zero, and we use R_f and r_f to define KOLs' reviews with and without observing MR, respectively. R_f is influenced by two factors: the expectation of KOLs' perception μ and MR r_m if revealed (i.e., $R_f = r_m + E\left(\mu\right)$), whereas r_f is only influenced by the perception of crowdsourcing designs μ , uniformly distributed over $\left[-\tau,\tau\right]$. The specific analysis can be seen in the following section.

If the brand enterprise has the MR r_m ($s_A=a$) and decides to share it with KOLs and prospective consumers ($s_S=s$), KOLs will observe r_m ; hence KOLs' FR is scaled as $R_f=r_m+E\left(\mu\right)=r_m$, due to $E\left(\mu\right)=0$. However, if the brand enterprise does not obtain MR, it has no information to deliver to KOLs; thus, prospective consumers will make purchase decisions solely based on FR (r_t).

Following prior work (Li et al., 2021), the expression of JR is denoted by $r_{_{\! J}}=r_{_{\! m}}+\lambda R_{_{\! f}}$, where λ represents consumers' sensitivity to KOLs' reviews ($\lambda>0$); the JR impacts the consumers' purchase intentions in conjunction with price. The reason is that better comments bring more sales and further induce the enterprise to increase the price. In this way, prospective consumers will decide whether to order crowdsourcing products via JR.

Meanwhile, the acquisition of online reviews incurs costs and assumes that the brand enterprise's information acquisition cost from the manufacturer is c_a , uniformly distributed over $\left[0,l\right]$ with density function $h\left(\cdot\right)$; it varies with the degree of precision and accuracy of MR information. Similar to the related literature for mathematical tractability, production costs are normalized to zero.

Table 2 presents the summary of the notation used in this paper.

Figure 1 demonstrates the timeline of the model. The enterprise decides whether to acquire MR from the manufacturer in the first stage. In the second stage, if it obtains MR, the enterprise makes the MR-sharing decision. In stage three, whether the enterprise shares the MR or not, KOLs always post their FR. In stages four and five, influenced by the JR, potential consumers make the order decision; meanwhile, the enterprise makes the production/pricing decision based on the expected profit of the crowdsourcing products, and fulfills the orders. Otherwise, the crowdsourcing design is abandoned. We use backward induction to solve the problem, detailed in the following section.

Table 2. Summary of notations

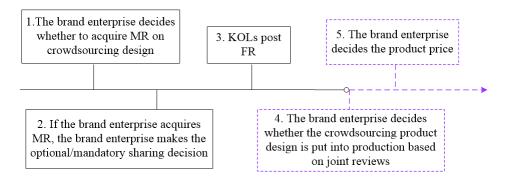
Notation	Explanation		
r_m	Manufacturer's reviews (MR), uniformly distributed over $\left(0,\overline{r}_{_{\!m}}\right]$ with the distribution function $F\left(\cdot\right)$ and the density function $f\left(\cdot\right)$		
R_{f}	KOLs' reviews (FR) when observing MR		
$r_{_f}$	KOLs' reviews (FR) when not observing MR		
9	Consumers' recognition level		
μ	KOLs' perception of crowdsourcing designs, uniformly distributed over $[- au, au]$		
c_{a}	MR acquisition cost from the manufacturer, uniformly distributed over $\left[0,l\right]$		
$s_{\scriptscriptstyle A}$	Brand enterprise's information acquisition scheme ($s_{\scriptscriptstyle A} \in \{a, na\}$)		
q	Q Crowdsourcing design quality level		
$q_{ m max}$	The highest crowdsourcing design quality level		
l	The upper bound of the acquisition cost		
$\overline{r}_{_{m}}$	The highest manufacturer's MR		
$r_{_{\!J}}$	Joint reviews from the manufacturer and KOLs		
p	Crowdsourcing product selling price		
$p_{_w}$	Crowdsourcing product wholesale price		
Q	Crowdsourcing product demand		
$a_{_0}$	The basic recognition level		
λ	Consumers' sensitivity to KOLs' FR		
$s_{_S}$	Brand enterprise's sharing scheme ($s_{_{S}} \in \left\{s, ns\right\}$)		
b	The likelihood of the brand enterprise acquiring MR information		
π	π Brand enterprise's post profit		

continued on following page

Table 2. Continued

Notation	Explanation	
$E[\pi]$	Brand enterprise's prior profit	
u_{cs}	Expected consumer surplus	
~	Optional sharing	
^	Mandatory sharing	

Figure 1. Sequence of events



ANALYSIS

In this section, we first analyze the two scenarios where the enterprise acquires MR ($s_{\scriptscriptstyle A}=a$) or does not acquire MR from the manufacturer ($s_{\scriptscriptstyle A}=na$), and then solve the enterprise's optimal MR acquisition decision, $s_{\scriptscriptstyle A}$.

We use the backward method for each scenario to derive the optimal solution for the five-stage problems defined in Figure 1. Specifically, at stages 5 and 4, we identify the brand enterprise's optimal price given the MR acquisition/sharing decision and KOLs' FR level. At stage 3, we solve KOLs' optimal FR level. In stage 2, we identify the brand enterprise's optimal sharing decision. At stage 1, we examine the brand enterprise's optimal MR acquisition decision and finally obtain the optimal information acquisition and sharing decisions.

Scenario One: The Brand Enterprise Acquires MR ($s_A = a$)

In this scenario, the enterprise decides to acquire MR from the manufacturer. We can derive its optimal price at the last stage by maximizing its profit $\pi = \left(p - p_w\right)Q - c_a = \left(p - p_w\right)\left[\left(\vartheta - p\right)/\xi\right] - c_a$ based on $\partial \pi / \partial p = 0$, so $p^* = \left(\vartheta + p_w\right)/2$. Thus, the brand enterprise's optimal profit is

$$\boldsymbol{\pi}^{*} = \left(p - p_{_{\boldsymbol{w}}}\right)Q - c_{_{\boldsymbol{a}}} = \left[\left(\vartheta - p_{_{\boldsymbol{w}}}\right)/\left.2\right] \cdot \left\{\left[\vartheta - \left(\left(\vartheta + p_{_{\boldsymbol{w}}}\right)/\left.2\right)\right]/\left.\xi\right\} - c_{_{\boldsymbol{a}}} = \left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)^{2}/\left.4\xi - c_{_{\boldsymbol{a}}}\right\}\right\} - c_{_{\boldsymbol{a}}} = \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right] \cdot \left\{\left[\vartheta - \left(\left(\vartheta + \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right)\right]\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right]\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right]\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right]\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right]\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right]\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right]\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right]\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right]\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right] + \left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right] + \left[\left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right] + \left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right] + \left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right] + \left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right] + \left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right] + \left(\vartheta - \boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left(\vartheta - \boldsymbol{p}_{$$

After acquiring the MR, the enterprise has two sharing schemes: the optional sharing scheme and the mandatory sharing scheme.

Optional Sharing Scheme

Under the optional sharing scheme, the brand enterprise can choose to reveal or hide the manufacturer's MR to KOLs, and the related decision is denoted by $s_{\scriptscriptstyle S} \in \left\{s,ns\right\}$. If the brand enterprise shares the information with KOLs ($s_{\scriptscriptstyle S}=s$), FR $R_{\scriptscriptstyle f}$ equals to $r_{\scriptscriptstyle m}$; thus consumers' recognition level is $\vartheta=\tilde{\vartheta}^a_{\scriptscriptstyle s}=\left(1+a_{\scriptscriptstyle 0}\right)\left(r_{\scriptscriptstyle m}+\lambda r_{\scriptscriptstyle m}\right)$. Therefore, the optimal price charged by the enterprise is $p^*=\tilde{p}^a_{\scriptscriptstyle s}=\left[\left(1+a_{\scriptscriptstyle 0}\right)\left(r_{\scriptscriptstyle m}+\lambda r_{\scriptscriptstyle m}\right)+p_{\scriptscriptstyle w}\right]/2$. Then we obtain the enterprise's profit:

$$\tilde{\pi}_{\scriptscriptstyle s}^{\scriptscriptstyle a} = [\left(1+a_{\scriptscriptstyle 0}\right)\!\left(r_{\scriptscriptstyle m}+\lambda r_{\scriptscriptstyle m}\right)\!-p_{\scriptscriptstyle w}]^{\scriptscriptstyle 2} \; / \; 4\xi - c_{\scriptscriptstyle a}$$

If the enterprise decides not to share r_m with KOLs ($s_S=ns$), KOLs will make independent and rational inferences from the enterprise's non-sharing behavior; thus they will post their FR r_f . Therefore, $\vartheta=\tilde{\vartheta}^a_{ns}=\left(1+a_0\right)\left(r_m+\lambda r_f\right)$, and the enterprise will charge the optimal price $p^*=\tilde{p}^a_{ns}=\left[\left(1+a_0\right)\left(r_m+\lambda r_f\right)+p_w\right]/2$. Under this circumstance, the enterprise's profit can be written as $\tilde{\pi}^a_{ns}=\left[\left(1+a_0\right)\left(r_m+\lambda r_f\right)-p_w\right]^2/4\xi-c_a$.

The comparison between $\tilde{\pi}_s^a$ and $\tilde{\pi}_{ns}^a$ offers the condition under which the enterprise is willing to convey the manufacturer's comments to KOLs, producing the following lemma.

Lemma 1: When the enterprise acquires MR, it intends to choose to share MR with KOLs to influence the prospective consumers if and only if $r_m > r_f$.

Lemma 1 shows that the enterprise would share MR with KOLs when acquiring and disclosing MR is remunerative. It also means that sharing MR should help improve FR on crowdsourcing designs to some degree. Otherwise, the enterprise will not share MR.

Additionally, if KOLs do not obtain MR from the enterprise, KOLs do not know if the enterprise has obtained MR but hid it or if the enterprise has not obtained MR and therefore cannot reveal it. To this end, there are two situations in the setting of the enterprise's non-sharing behavior, and we use probabilities to describe each situation: (1) given the MR information acquisition cost, $P(s_A=a)=b\;, (0< b<1)\;, \text{ which describes the probability that the enterprise seeks the MR. Hence, the likelihood of not obtaining MR information is <math display="inline">1-b\;, \text{i.e.},\; P(s_A=na)=1-b\;; (2)$ if the enterprise has obtained MR (i.e., $r_m< r_f$, based on Lemma 1) but hidden it, the probability is $P(s_A=a,r_m\leq r_f)=bF\left(r_f\right).$ We can identify the conditional probability in two situations upon the enterprise's non-sharing:

$$P(s_{_{A}} = na \mid s_{_{S}} = ns) = (1-b)/[(1-b)+bF(r_{_{f}})]$$

and:

$$P(s_{A} = a \mid s_{S} = ns) = bF(r_{f}) / \left[\left(1 - b\right) + bF(r_{f}) \right]$$

Thus, FR's expectation of the enterprise's non-sharing is

$$r_{\scriptscriptstyle f} = \frac{1-b}{\left(1-b\right)+bF\left(r_{\scriptscriptstyle f}\right)} \cdot \left(\overline{r}_{\scriptscriptstyle m} \mathrel{/} 2\right) + \frac{bF\left(r_{\scriptscriptstyle f}\right)}{\left(1-b\right)+bF\left(r_{\scriptscriptstyle f}\right)} \cdot \left(r_{\scriptscriptstyle f} \mathrel{/} 2\right).$$

In the situation when $s_{_A}=na\mid s_{_S}=ns$, KOLs only know the crowdsourcing design quality distribution, hence inferring the expected design quality $\overline{r}_{_m} \ / \ 2$. As a result, KOLs' FR is $R_{_f}=\left(\overline{r}_{_m} \ / \ 2\right)+E\left(\mu\right)=\overline{r}_{_m} \ / \ 2$.

In the situation when $s_A=a\mid s_S=ns$, according to Lemma 1, KOLs know that the acquired MR is lower than r_f , implying that the crowdsourcing designs' manufacturability is undesirable, which leads to lower KOLs' comments on the crowdsourcing designs; it reflects their FR level is uniformly distributed over $\left(0,r_f\right]$ rather than $\left(r_f,+\infty\right)$. In this regard, where $s_A=a\mid s_S=ns$, the expected value of KOLs toward crowdsourcing design quality is $r_f\neq 0$ and $R_f=\left(r_f\neq 0\right)+E\left(\mu\right)=r_f\neq 0$.

Thus, the value of KOLs' review level (FR) r_t under the enterprise's non-sharing is as follows,

$$r_f = \frac{\overline{r}_m \left| \sqrt{1-b} - \left(1-b\right) \right|}{b}.$$
 It is clear that r_f decreases with b , indicating that under the enterprise's non-sharing, the higher the likelihood of MR information acquisition is for the enterprise, and the lower FR level would be posted by KOLs. When $b=1$, it implies that when the enterprise acquires but hides MR, KOLs may infer that the enterprise's unwillingness to reveal it is because the MR is low, which could negatively impact FR. On the other hand, if $b=0$, it implies that the enterprise does not acquire MR, and KOLs' FR r_f is $\overline{r}_m/2$, which is reasonable as KOLs would not know the manufacturability of crowdsourcing designs, rather they could only infer it from the manufacturer's MR distribution function.

Hence, we derive $0 < r_f < q \overline{f}_{_{max}}$. Consider the different sharing situations under the enterprise's acquisition of MR; the enterprise's profit is $\tilde{\pi}_a = \int_{-0}^{r_f} (\tilde{\pi}_{ns}^a) dF\left(r_m\right) + \int_{-r_f}^{\overline{r}_m} (\tilde{\pi}_s^a) dF\left(r_m\right)$, where the first term of the right side of the equation is profit under the non-sharing condition, and the second term is the profit under the sharing condition for r_m uniformly distributed over $\left(0,\overline{r}_m\right]$ with the distribution function $F\left(\cdot\right)$ and the density function $f\left(\cdot\right)$.

Mandatory Sharing Scheme

In this setting, the MR acquired by the brand enterprise must be shared publicly with KOLs and prospective consumers. Thereby, given the MR $r_{_m}$, FR is $R_{_f}=r_{_m}$ due to $R_{_f}=r_{_m}+E\left(\mu\right)$ where $E\left(\mu\right)=0$. Under the impact of JR, the consumers' recognition level is $\vartheta=\hat{\vartheta}_{_s}^a=\left(1+a_{_0}\right)\left(r_{_m}+\lambda r_{_m}\right)$. Table 3 lists all cases of the consumers' recognition levels.

With the profit function, we can derive the optimal product selling price $p^* = \hat{p}_s^a = \left[\left(1+a_0\right)\left(r_m + \lambda r_m\right) + p_w\right]/2$, and the enterprise's optimal profit is:

$$\hat{\pi}_{s}^{a} = [(1 + a_{0})(r_{m} + \lambda r_{m}) - p_{m}]^{2} / 4\xi - c_{a}$$

	Consumer reco $\vartheta = \left(1 + \frac{1}{2}\right)$	υ ,	The probability of acquiring MR $\label{eq:probability} \mathrm{P}(s_{{\scriptscriptstyle A}}=a)=b$	The probability of non-acquiring $$\operatorname{MR}$$ $P(s_{{}_{\!A}}=na)=1-b$
	Optional sharing	sharing	$ec{artheta} = ilde{artheta}_s^a = igl(1 + a_{_0}igr)igl(r_{_m} + \lambda r_{_m}igr)$	$artheta = ilde{artheta}_s^{na} = ig(1+a_{_0}ig)*\lambda r_{_f}$
	scheme	No sharing	$artheta = ilde{artheta}_{ns}^a = igl(1 + a_0igr)igl(r_{\!\!\!\ m} + \lambda r_{\!\!\!\ f}igr)$	$artheta = ilde{artheta}_{ns}^{na} = ig(1+a_{_0}ig)*\lambda r_{_f}$
	Mandatory sharing scheme	sharing	$artheta = \hat{artheta}_s^a = igl(1 + a_0igr)igl(r_{\!\scriptscriptstyle m} + \lambda r_{\!\scriptscriptstyle m}igr)$	$artheta=\hat{artheta}_{s}^{na}=\left(1+a_{_{0}} ight)*\lambda\left(\overline{r}_{_{m}}\;/\;2 ight)$

Table 3. Summary of the consumer recognition level

Since r_m uniformly distributes over $\left(0,\overline{r}_m\right]$ with the distribution function $F\left(\cdot\right)$ and the density function $f\left(\cdot\right)$, the enterprise's profit under the mandatory sharing and acquisition case is $\hat{\pi}_a = \int_0^{\overline{r}_m} (\hat{\pi}_s^a) dF\left(r_m\right)$

The above analysis focuses on two sharing schemes in the MR acquisition scenario. Next, we will highlight those in the non-acquisition scenario.

Scenario Two: The Enterprise Does Not Acquire MR ($s_{_{A}}=na$)

In this scenario, the enterprise chooses not to obtain MR, and the enterprise's optimal selling price can be calculated by maximizing its profit $\pi = \left(p - p_{_w}\right)Q$, where $Q = \left(\vartheta - p\right)/\xi$, namely, $p^* = \left(\vartheta + p_{_w}\right)/2$ from the first-order condition $\partial \pi/\partial p = 0$. Thus, the enterprise's optimal profit is:

$$\boldsymbol{\pi}^{*} = \left[\left(\vartheta - \boldsymbol{p}_{\scriptscriptstyle \boldsymbol{w}} \right) / \, 2 \right] \cdot \left\{ \left[\vartheta - \left(\left(\vartheta + \boldsymbol{p}_{\scriptscriptstyle \boldsymbol{w}} \right) / \, 2 \right) \right] / \, \boldsymbol{\xi} \right\} = \left(\vartheta - \boldsymbol{p}_{\scriptscriptstyle \boldsymbol{w}} \right)^{2} \, / \, 4 \boldsymbol{\xi}$$

In the non-acquisition scenario, the enterprise has two sharing schemes: optional sharing and mandatory sharing.

Optional Sharing Scheme

In this scenario, the enterprise has no MR to share with KOLs and prospective consumers, so KOLs cannot see MR, but they will make rational inferences from the enterprise's non-sharing behavior under the optional sharing scheme, that is, FR is r_f . Thus, the impact of JR is transferred into one single review, FR alone, and the prospective consumers' recognition level is $\vartheta = \tilde{\vartheta}_{ns}^{na} = \left(1+a_0\right)*\lambda r_f$. Under this circumstance, the optimal product selling price charged by the enterprise is $p^* = \tilde{p}_{ns}^{na} = \left[\left(1+a_0\right)*\lambda r_f + p_w\right]/2$, and the enterprise's optimal profit can be derived as $\tilde{\pi}_{ns}^{na} = \left[\lambda\left(1+a_0\right)r_f - p_w\right]^2/4\xi$. Therefore, in the non-acquisition context, the enterprise's profit under the optional sharing scheme is $\tilde{\pi}_{na} = \int_{0}^{\tilde{r}_m} (\tilde{\pi}_{ns}^{na}) dF\left(r_m\right)$.

Mandatory Sharing Scheme

Similarly, under the mandatory sharing scheme, the KOLs can only deduce from the distribution function of crowdsourcing design quality due to the enterprise's no-acquisition behavior; that is,

KOLs' assessed crowdsourcing design product quality is the expected value of crowdsourcing design quality and KOLs' FR $R_f = \left(\overline{r}_m \ / \ 2\right) + E\left(\mu\right) = \overline{r}_m \ / \ 2$. JR merely contains FR; thus, the consumers' recognition level is $\vartheta = \hat{\vartheta}_{ns}^{na} = \left(1 + a_0\right) * \lambda \left(\overline{r}_m \ / \ 2\right)$. The optimal product price charged by the enterprise is $p^* = \hat{p}_{ns}^{na} = \left[\left(1 + a_0\right) * \lambda \left(\overline{r}_m \ / \ 2\right) + p_w\right] / 2$, and the enterprise's optimal profit is $\hat{\pi}_{ns}^{na} = \left[\lambda \left(1 + a_0\right) \left(\overline{r}_m \ / \ 2\right) - p_w\right]^2 / 4\xi$. Given that the enterprise's MR information sharing scheme is predetermined, in the non-acquisition context, the enterprise's profit under the mandatory sharing scheme is $\hat{\pi}_{na} = \int_0^{\overline{r}_m} (\hat{\pi}_{ns}^{na}) dF(r_m)$.

Based on the MR acquisition and non-acquisition, we proceed to analyze the optional sharing scheme. The enterprise chooses to acquire MR if and only if $\tilde{\pi}_a \geq \tilde{\pi}_{na}$, which can be reduced to the condition $c_a \leq \tilde{l}$, where:

$$\begin{split} \tilde{l} &= \{ \left(1 + a_{\scriptscriptstyle 0}\right) [\left(\overline{r}_{\scriptscriptstyle m} + 2r_{\scriptscriptstyle f}\right) \left(1 + a_{\scriptscriptstyle 0}\right) (\overline{r}_{\scriptscriptstyle m} - r_{\scriptscriptstyle f})^2 \lambda^2 + (\left(2a_{\scriptscriptstyle 0} + 2\right) \overline{r}_{\scriptscriptstyle m}^{ 3} - 3 \overline{r}_{\scriptscriptstyle m}^{ 2} p_{\scriptscriptstyle w} \\ &+ 6 \overline{r}_{\scriptscriptstyle m} r_{\scriptscriptstyle f} p_{\scriptscriptstyle w} + r_{\scriptscriptstyle f}^{ 2} \left(\left(1 + a_{\scriptscriptstyle 0}\right) r_{\scriptscriptstyle f} - 3 p_{\scriptscriptstyle w}\right) \lambda + \overline{r}_{\scriptscriptstyle m}^{ 2} \left(\overline{r}_{\scriptscriptstyle m} \left(1 + a_{\scriptscriptstyle 0}\right) - 3 p_{\scriptscriptstyle w}\right)] \} \, / \, 12 \xi \overline{r}_{\scriptscriptstyle m} \end{split}$$

Due to the likelihood of the enterprise's acquisition of MR $b=P(c_a\leq \tilde{l})$, the optimal acquisition and sharing scheme is shown in the following theorem.

Theorem 1: Under the optional sharing scheme, the enterprise acquires MR on crowdsourcing designs if and only if $c_a \leq \tilde{l}$, and reveals it to KOLs when $r_m > r_f$; vice versa.

Theorem 1 indicates that the enterprise will acquire and share MR only if its acquisition cost is relatively low and the MR is higher than KOLs' comments on crowdsourcing designs. It implies that when the ratio of benefit to expenditure from MR is higher, the brand enterprise tends to acquire and share them, which yields the JR on crowdsourcing designs; otherwise, the process of information acquisition and sharing will stop halfway or even totally.

Next, we further investigate the enterprise's acquisition in the mandatory sharing scheme, where the enterprise is mandated to share the acquired MR. The outcome is that the enterprise chooses to acquire MR information if and only if $\hat{\pi}_a \geq \hat{\pi}_{na}$, which can be reduced to the condition, $c_a \leq \hat{l}$,

where
$$\hat{l}=\frac{\overline{r}_{_{m}}\left(1+a_{_{0}}\right)\left[\overline{r}_{_{m}}\left(1+a_{_{0}}\right)\left(\lambda^{2}+8\lambda+4\right)-12p_{_{w}}\right]}{48\xi}$$
 . Therefore, we have the following theorem.

Theorem 2: Under the mandatory sharing scheme, the enterprise acquires MR on crowdsourcing designs if and only if $c_a \leq \hat{l}$, and vice visa.

Similar to Theorem 1, Theorem 2 also demonstrates that, under the mandatory sharing, only if the enterprise's acquisition cost is relatively low, it is inclined to acquire MR. This means that, regardless of the sharing schemes, the enterprise's acquisition scheme is strongly related to the corresponding costs.

Recalling Theorems 1 and 2, we now compare two costs: \tilde{l} and \hat{l} , and analyze how the brand enterprise's acquisition of MR information changes with the optional/mandatory sharing scheme. The possibility of the brand enterprise acquiring MR under the mandatory and optional sharing are $P(c_a \leq \hat{l}) = \hat{l} / l$ and $P(c_a \leq \tilde{l}) = \tilde{l} / l$, respectively. Under the mandatory sharing, we can obtain:

$$\begin{split} &P(c_{a} \leq \tilde{l}) - P(c_{a} \leq \hat{l}) = \left(\tilde{l} - \hat{l}\right) / \, l \\ &= \frac{\left(1 + a_{0}\right) \lambda \left[\left(3\overline{r_{m}}^{3}\lambda - 12\overline{r_{m}}\lambda r_{f}^{2} + 8\lambda r_{f}^{3} + 4r_{f}^{3}\right) \left(1 + a_{0}\right) - 12\,p_{w}\left(\overline{r_{m}} - r_{f}\right)^{2}\right]}{48\overline{r_{m}}\xi l} \end{split}$$

$$\text{By solving } P(c_{_{a}} \leq \widetilde{l} \,) > P(c_{_{a}} \leq \widehat{l} \,) \text{ , we obtain } \lambda > \frac{12 p_{_{w}} (\overline{r}_{_{m}} - r_{_{f}})^2 - 4 \left(1 + a_{_{0}}\right) r_{_{f}}^{\ 3}}{\left(3 \overline{r}_{_{m}}^{\ 3} - 12 \overline{r}_{_{m}} r_{_{f}}^{\ 2} + 8 r_{_{f}}^{\ 3}\right) \left(1 + a_{_{0}}\right)}, \text{ which reflects}$$

the possibility of the enterprise's acquiring MR under the optional sharing is higher than the mandatory sharing if $\lambda > \frac{12 p_{_w} (\overline{r}_{_m} - r_{_f})^2 - 4 \left(1 + a_{_0}\right) r_{_f}^3}{\left(3\overline{r}_{_m}^{\ 3} - 12\overline{r}_{_m} r_{_f}^{\ 2} + 8 r_{_f}^{\ 3}\right) \left(1 + a_{_0}\right)}$. Therefore, we can derive the following Lemma.

Lemma 2: The enterprise is more likely to acquire MR on crowdsourcing design with the optional sharing than the mandatory sharing scheme if $\lambda > \frac{12 p_{_w} (\overline{r}_{_m} - r_{_f})^2 - 4 \left(1 + a_{_0}\right) r_{_f}^3}{\left(1 + a_{_0}\right) \left(3\overline{r}_{_m}^{\ 3} - 12\overline{r}_{_m}r_{_f}^2 + 8r_{_f}^{\ 3}\right)}$, and vice versa.

Lemma 2 implies that the enterprise has more motivation to obtain MR under the optional sharing scheme when potential consumers are more sensitive to FR; this further verifies that the enterprise prefers to implement JR on crowdsourcing designs in an open online environment. In the optional sharing, even if the enterprise does not acquire MR, KOLs can infer whether the MR is good or not, and this will decrease the FR-sensitive consumers' purchase uncertainties.

Lemma 3: The enterprise under the optional sharing scheme is more inclined to share MR on crowdsourcing design with KOLs than under the mandatory sharing if $0 < \frac{r_f}{\overline{r}_m} < 1 - \frac{\hat{l}}{\tilde{l}}$, and vice versa.

Lemma 3 indicates that the enterprise is inclined to select the optional sharing scheme to disclose MR instead of the mandatory sharing when the ratio of FR to the highest MR is below a certain threshold value and vice versa. When there is a significant discrepancy between them (i.e., below a certain value), it leads to potential consumers' skeptical and wait-and-see attitude toward purchasing crowdsourced products, thus impeding the firm to select the mandatory sharing scheme for revealing MR, and vice versa. It suggests that the firm should strengthen communication between KOLs and the manufacturer in the crowdsourcing process, thus narrowing their divergence in comments and improving crowdsourcing quality. In practical cases, such as Xiaomi, by strengthening communication and interaction with fans (KOLs) and manufacturers, crowdsourcing designs can be continuously improved before production without relatively-wide gap comments.

Impact on Profits and Consumer Surplus

Impact on Profits

Having derived the enterprise's optimal MR acquisition in two sharing schemes, we next explore how the two MR sharing schemes impact the enterprise's post-profit (after the MR acquisition cost is realized) and the prior profit (before the MR acquisition cost is realized). First, the enterprise's post-profit is analyzed.

Post-profit. When the enterprise acquires MR ($s_A = a$), recall the optimal outcome in Theorem 1; the enterprise's post-profit displays two patterns under two sharing schemes, exhibited below.

Under the optional sharing scheme, the enterprise acquires MR when $c_a \in \left[0, \tilde{l}\right]$. Thus, its post-profit, when the cost of MR information acquisition is realized, is:

$$\tilde{\pi} = \underbrace{\int_{0}^{r_{f}} \frac{\left[\left(1 + a_{0}\right)\left(r_{m} + \lambda r_{f}\right) - p_{w}\right]^{2}}{4\xi} dF(r_{m})}_{\text{Non-sharing}} + \underbrace{\int_{r_{f}}^{\overline{r}_{m}} \frac{\left[\left(1 + a_{0}\right)\left(r_{m} + \lambda r_{m}\right) - p_{w}\right]^{2}}{4\xi} dF(r_{m})}_{\text{Sharing}} dF(r_{m}) - c_{a}. \tag{1}$$

Under the mandatory sharing scheme, the enterprise acquires MR information when $c_a \in [0,\hat{l}\,]$. Then its post-profit is:

$$\hat{\pi} = \int_{0}^{\bar{r}_{m}} \frac{\left[\left(1 + a_{0} \right) \left(r_{m} + \lambda r_{m} \right) - p_{w} \right]^{2}}{4\xi} dF(r_{m}) - c_{a}$$
(2)

When the enterprise does not acquire MR ($s_A = na$), following the same logic as above, its post-profit under the optional sharing is:

$$\tilde{\pi} = \frac{\left[\lambda \left(1 + a_0\right) r_f - p_w\right]^2}{4\xi} \tag{3}$$

and its post-profit under the mandatory sharing scheme is:

$$\hat{\pi} = \frac{\left[\lambda \left(1 + a_0\right) \left(\overline{r}_m / 2\right) - p_w\right]^2}{4\varepsilon} \tag{4}$$

We obtain the following theorem by comparing the enterprise's post-profits under two sharing schemes.

Theorem 3: The enterprise's post-profit under the optional sharing is higher than the mandatory sharing:

(1) if
$$c_a < F$$
 and $\lambda > \max\{\lambda_1, \left(3p_w - r_f - a_0r_f\right) / \left[2r_f\left(1 + a_0\right)\right]\}$; or

(2) if
$$\left(3p_{_w}-r_{_f}-a_{_0}r_{_f}\right)/\left[2r_{_f}\left(1+a_{_0}\right)\right]<\lambda<\lambda_{_1}\,\mathrm{and}~c_{_a}<\tilde{l}$$
 , where:

$$F = \left\{ \! \left(\! 1 + a_{\scriptscriptstyle 0} \right) \! \left[\! \frac{ \left(\! \left(1 + a_{\scriptscriptstyle 0} \right) \overline{r_{\scriptscriptstyle m}}^{\,\, 3} + 8 r_{\scriptscriptstyle f}^{\,\, 3} \left(1 + a_{\scriptscriptstyle 0} \right) \right) \lambda^{2} \right. \\ \left. + \left(\! \left(8 a_{\scriptscriptstyle 0} + 8 \right) \overline{r_{\scriptscriptstyle m}}^{\,\, 3} + 4 r_{\scriptscriptstyle f}^{\,\, 2} \left(a_{\scriptscriptstyle 0} r_{\scriptscriptstyle f} + r_{\scriptscriptstyle f} - 3 p_{\scriptscriptstyle w} \right) \right) \lambda \right] \right\} / \left. 48 \xi \overline{r_{\scriptscriptstyle m}} \right. \\ \left. + 4 \left(\! \left(1 + a_{\scriptscriptstyle 0} \right) \overline{r_{\scriptscriptstyle m}} - 3 p_{\scriptscriptstyle w} \right) \overline{r_{\scriptscriptstyle m}}^{\,\, 2} \right. \right.$$

Theorem 3 reveals the different areas of acquisition cost and consumers' sensitivity to FR, in which the enterprise is more profitable under the optional/mandatory sharing scheme. Specifically, when the MR acquisition cost is relatively low, and the level of consumers' sensitivity to FR is high and medium, it encourages the enterprise to acquire and share MR under the optional sharing scheme.

The rationale behind it is that when MR acquisition cost is small, this triggers the enterprise to obtain MR, and then the optional sharing enables the enterprise to avoid revealing the lower-level MR to KOLs, thus benefiting the enterprise. Conversely, when the MR acquisition cost is high, it impedes the enterprise from acquiring MR; the optional sharing leads to a lower FR and harms the enterprise. In this regard, the enterprise prefers the mandatory sharing to reduce the loss risk.

Prior profit: For the enterprise's prior profits, under the optional sharing, if $c_a \leq \tilde{l}$, the enterprise's post-profit is illustrated by formula (1); if $\tilde{l} < c_a \leq l$, its post-profit is formula (3), so the enterprise's prior profit is formulated as follows:

$$E\left[\tilde{\pi}\right] = H\left(\tilde{l}\right) \left[\int_{0}^{r_{f}} \frac{\left[\left(1+a_{0}\right)\left(r_{m}+\lambda r_{f}\right)-p_{w}\right]^{2}}{4\xi} dF\left(r_{m}\right) \right] \\ + H\left(\tilde{l}\right) \left[\int_{r_{f}}^{\bar{r}_{m}} \frac{\left[\left(1+a_{0}\right)\left(r_{m}+\lambda r_{m}\right)-p_{w}\right]^{2}}{4\xi} dF\left(r_{m}\right) \right] \\ + \left[1-H\left(\tilde{l}\right)\right] \cdot \frac{\left[\lambda\left(1+a_{0}\right)r_{f}-p_{w}\right]^{2}}{4\xi} - \int_{0}^{\bar{l}} c_{a} dH\left(c_{a}\right) \\ \frac{No acquisition}{4\xi} - \int_{0}^{$$

Likewise, under the mandatory sharing scheme, if $c_a \leq \hat{l}$ the enterprise's post-profit is given by formula (2); if $\hat{l} < c_a \leq l$, its post-profit is formula (4). Then, the enterprise's prior profit can be written as:

$$\begin{split} E\left[\hat{\pi}\right] &= H\left(\hat{l}\right) \left[\int_{0}^{\overline{r}_{m}} \frac{\left[\left(1+a_{0}\right)\left(r_{m}+\lambda r_{m}\right)-p_{w}\right]^{2}}{4\xi} dF\left(r_{m}\right) \right] \\ &+ \left[1-H\left(\hat{l}\right)\right] \cdot \frac{\left[\lambda\left(1+a_{0}\right)\left(\overline{r}_{m}/2\right)-p_{w}\right]^{2}}{4\xi} - \int_{0}^{\hat{l}} c_{a} dH\left(c_{a}\right) \end{split} \tag{6}$$

By comparing the results, the prior profit of the enterprise under the optional sharing scheme and the mandatory sharing scheme generates the following theorem.

Theorem 4: The enterprise's prior profit under the mandatory sharing is higher than under the optional sharing if $l > \left(\frac{1}{2} + A\right)(\tilde{l} - \hat{l})$, and vice versa, where:

$$A = \frac{{{{\left[{{{\left({1 + {a_0}} \right)}{{\left({4r_f}^3 + \overline r_m^{\ 3}} \right)} \right]}{\lambda ^2}} + \left[{8\left({1 + {a_0}} \right)\overline r_m^{\ 3} + 2r_f^{\ 2}\left({{a_0}{r_f} - 3{p_w} + {r_f}} \right)} \right]\lambda + 4\overline r_m^{\ 2}\left[{{{\left({1 + {a_0}} \right)}\overline r_m - 3{p_w}} \right]}}{{3\lambda \overline r_m\left({\overline r_m - 2{r_f}} \right)}{\left[{\lambda \left({1 + {a_0}} \right)\left({\overline r_m + 2{r_f}} \right) - 4{p_w}} \right]}}}$$

Theorem 4 shows that, from the prior-profit lens, the enterprise tends to choose the mandatory sharing instead of the optional sharing when the upper bound of the acquisition cost is relatively

higher. Recall that the enterprise's prior profit is the sum of the post-profits in every possible MR acquisition cost, and it is observed that the optional sharing could lead to a higher post-profit when the MR acquisition cost is at a low level compared with the mandatory sharing. If the upper bound of the acquisition cost increases (i.e., is relatively higher), this means the feasible acquisition cost also increases, thus the enterprise is more likely to acquire MR. Meanwhile, non-sharing MR under the optional sharing scheme leads to a decrease in KOLs' valuation expectations $r_f < \overline{r}_m / 2$. By contrast, the enterprise's non-sharing behavior under the mandatory sharing will give rise to KOLs' valuation expectations as $\overline{r}_m / 2$, under this situation, the optional sharing scheme ruins consumers' expectations compared with under the mandatory sharing, thus the enterprise prefers to adopt the mandatory sharing scheme.

Impact on Consumer Surplus

This subsection investigates how consumer surplus changes under different acquisition/sharing schemes. When the enterprise acquires MR ($s_A=a$), we derive the consumer surplus given by $\widetilde{CS}_s^a=[\left(1+a_0\right)\left(r_m+\lambda r_m\right)-p_w]^2/8\xi$ if the enterprise shares the acquired MR under the optional sharing. Otherwise, the consumer surplus is $\widetilde{CS}_{ns}^a=[\left(1+a_0\right)\left(r_m+\lambda r_f\right)-p_w]^2/8\xi$. Similar to the optional sharing, if the enterprise has acquired MR under the mandatory sharing, the consumer surplus is $\widehat{CS}_s^a=[\left(1+a_0\right)\left(r_m+\lambda r_m\right)-p_w]^2/8\xi$.

When the enterprise does not acquire MR ($s_{_A}=na$), the consumer surplus under the optional sharing is $\widetilde{CS}_{ns}^{na}=[\lambda\left(1+a_{_0}\right)r_{_f}-p_{_w}]^2 \ / \ 8\xi$. Under the mandatory sharing, if the enterprise does not acquire MR, the consumer surplus is $\widehat{CS}_{ns}^{na}=[\lambda(1+a_{_0})(\overline{r}_{_m}\ /\ 2)-p_{_w}]^2 \ / \ 8\xi$.

Therefore, we derive the expected consumer surplus under the optional sharing as:

$$\tilde{u}_{cs} = H(\tilde{l}) \left\{ \int_{0}^{r_{f}} \left[\frac{\left[\left(1 + a_{0} \right) \left(r_{m} + \lambda r_{f} \right) - p_{w} \right]^{2}}{8\xi} \right] dF(r_{m}) \right\} \\
+ H(\tilde{l}) \left\{ \int_{r_{f}}^{\bar{r}_{m}} \left[\frac{\left[\left(1 + a_{0} \right) \left(r_{m} + \lambda r_{m} \right) - p_{w} \right]^{2}}{8\xi} \right] dF(r_{m}) \right\} \\
+ \left[1 - H(\tilde{l}) \right] \cdot \frac{\left[\lambda \left(1 + a_{0} \right) r_{f} - p_{w} \right]^{2}}{8\xi} \right] dF(r_{m}) dF(r_{m})$$

The expected consumer surplus under the mandatory sharing is:

$$\hat{u}_{cs} = \underbrace{H\left(\hat{l}\right) \{ \int_{0}^{\overline{r}_{m}} \left[\frac{\left[\left(1+a_{0}\right)\left(r_{m}+\lambda r_{m}\right)-p_{w}\right]^{2}}{8\xi} \right] dF\left(r_{m}\right) \}}_{\text{No consistion}} + \underbrace{\left[1-H\left(\hat{l}\right)\right] \cdot \frac{\left[\lambda\left(1+a_{0}\right)\left(\overline{r}_{m} \ / \ 2\right)-p_{w}\right]^{2}}{8\xi}}_{\text{No consistion}} \tag{8}$$

By comparing the expected consumer surplus under the optional/mandatory sharing schemes, we have the following theorem.

Theorem 5: The expected consumer surplus is higher under the optional sharing than under the mandatory sharing if and only if:

$$l<\left\{A\lambda^{2}+4\lambda \widetilde{l}\left(\left(1+a_{_{0}}\right)r_{_{f}}^{\;3}-3p_{_{\boldsymbol{w}}}\left(\overline{r}_{_{\boldsymbol{m}}}-r_{_{\boldsymbol{f}}}\right)^{2}\right)+4\overline{r}_{_{\boldsymbol{m}}}^{\;2}\left(\overline{r}_{_{\boldsymbol{m}}}\left(1+2\lambda\right)\left(1+a_{_{\boldsymbol{0}}}\right)-3p_{_{\boldsymbol{w}}}\right)\!\left(\widetilde{l}-\widehat{l}\right)\!\right\}/\left.B^{2}+\left(2\lambda^{2}+4\lambda\widetilde{l}\left(\left(1+a_{_{\boldsymbol{0}}}\right)r_{_{\boldsymbol{0}}}^{\;3}-3p_{_{\boldsymbol{w}}}\left(\overline{r}_{_{\boldsymbol{m}}}-r_{_{\boldsymbol{f}}}\right)^{2}\right)+4\overline{r}_{_{\boldsymbol{m}}}^{\;2}\left(\overline{r}_{_{\boldsymbol{m}}}\left(1+2\lambda\right)\left(1+a_{_{\boldsymbol{0}}}\right)-3p_{_{\boldsymbol{w}}}\right)\!\left(\widetilde{l}-\widehat{l}\right)\!\right\}\right)$$

and vice versa, where $A=(4\left(\overline{r}_{\!\!\!m}+2r_{\!\!\!f}\right)\!\left(\overline{r}_{\!\!\!m}-r_{\!\!\!f}\right)^{\!2}\!\tilde{l}-\overline{r}_{\!\!\!m}^{3}\!\hat{l}\left)\!\left(1+a_{\scriptscriptstyle 0}\right)$ and:

$$B = 3\lambda \overline{r}_{\!{}_{\!{}^{m}}} \Big(\overline{r}_{\!{}_{\!{}^{m}}} - 2r_{\!{}_{\!{}^{m}}} \Big) \Big(\lambda \Big(\overline{r}_{\!{}_{\!{}^{m}}} + 2r_{\!{}_{\!{}^{m}}} \Big) \Big(1 + a_{\!{}_{\!{}^{0}}} \Big) - 4p_{\!{}_{\!{}^{m}}} \Big)$$

Theorem 5 indicates that consumers favor the optional sharing scheme over the mandatory sharing scheme when the upper bound of the acquisition cost is relatively lower. It is reasonable that individuals believe that the lower the upper bound of the acquisition cost is, the more savings the enterprise has for acquiring MR, and the cheaper consumers purchase crowdsourcing products will be. Meanwhile, the optional sharing enables the enterprise to obtain MR but hide undesirable MR, and vice versa.

FURTHER ANALYSIS

The research and analysis in sections 3 and 4 mainly address the enterprise's and KOLs' behaviors in JR. In this part, according to the characteristics of JR, we further analyze the impact of the change in the manufacturer's wholesale price ($p_w = k_1 \vartheta$; $p_w = k_2 q$) and consumers' sensitivity to fan reviews

(
$$\lambda_{\rm MR} > \lambda_{\rm non-MR}$$
 ; $\, \lambda_{\rm MR} < \lambda_{\rm non-MR}$).

Wholesale Pricing: $p_{w} = k_{1} \vartheta$

To reflect the fact that there is a positive correlation between the manufacturer's wholesale price $p_{_w}$ and market recognition level ϑ , assume $p_{_w}=k_{_1}\vartheta$. For this scenario, we have Theorem 6. Theorem 6:

- (a) Under the optional sharing scheme, the enterprise will acquire MR if and only if $c_a \leq \tilde{l}_1$, where $\tilde{l}_1 = \{(1-k_1)^2(1+a_0)^2\left(\overline{r}_m^{\ 3}\lambda^2 3\lambda^2r_f^{\ 2}\overline{r}_m + 2\lambda^2r_f^{\ 3} + 2\lambda\overline{r}_m^{\ 3} + \lambda r_f^{\ 3} + \overline{r}_m^{\ 3}\right)\}/12\xi\overline{r}_m$; under the mandatory sharing, the enterprise will acquire MR on crowdsourcing designs if and only if $c_a \leq \hat{l}_1$, where $\hat{l}_1 = \overline{r}_m^{\ 2}(1-k_1)^2(1+a_0)^2(\lambda^2+8\lambda+4)/48\xi$.
- (b) The enterprise's post-profit under the optional sharing is higher than the mandatory sharing if $c_a < \{(1-k_1)^2(1+a_0)^2\left(\overline{r_m}^3\lambda^2+8\lambda^2{r_f}^3+8\overline{r_m}^3\lambda+4\lambda{r_f}^3+4\overline{r_m}^3\right)\} \ /\ 48\xi\overline{r_m} \ , \ \text{and vice versa; the enterprise's prior profit under the mandatory sharing is higher than the optional sharing if <math display="block">l > \left(\frac{1}{2}+A_1\right)\left(\tilde{l_1}-\hat{l_1}\right) \ , \ \text{and vice versa, where:}$

$$A_{\!\scriptscriptstyle 1} = \left(\overline{r_{\!\scriptscriptstyle m}}^{\,\,3}\lambda^2 + 4\lambda^2 r_{\!\scriptscriptstyle f}^{\,\,3} + 8\overline{r_{\!\scriptscriptstyle m}}^{\,\,3}\lambda + 2\lambda r_{\!\scriptscriptstyle f}^{\,\,3} + 4\overline{r_{\!\scriptscriptstyle m}}^{\,\,3}\right)/\,3\overline{r_{\!\scriptscriptstyle m}}\lambda^2\left(\overline{r_{\!\scriptscriptstyle m}}^{\,\,2} - 4r_{\!\scriptscriptstyle f}^{\,\,2}\right)$$

$$(\textit{\textbf{k}}_{_{1}}=0.05~,~\textit{\textbf{a}}_{_{0}}=4~,~\lambda=1~,~\overline{r}_{_{\!\textit{\textbf{m}}}}=1~,~\xi=1~)~(\textit{\textbf{k}}_{_{2}}=0.2~,~\textit{\textbf{a}}_{_{0}}=4~,~\lambda=1~,~\overline{r}_{_{\!\textit{\textbf{m}}}}=1~,~\xi=1~)$$

Figure 2. The impact of $\,p_{_{\boldsymbol{w}}}=k_{_{\! 1}}\!\vartheta$

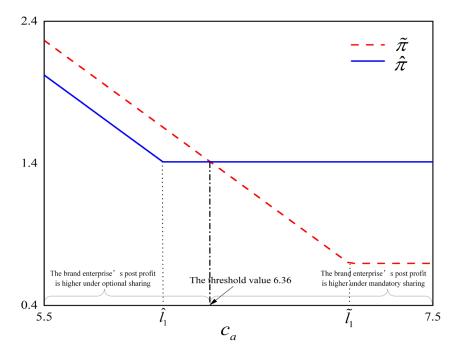
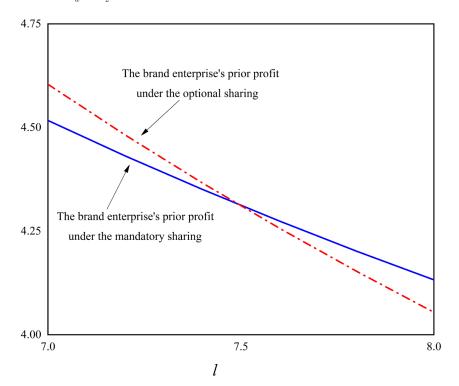


Figure 3. The impact of $\,p_{_{\boldsymbol{w}}}=k_{_{\!2}}q\,$



Theorem 6 and Figure 2 show a similar pattern as before, but the difference is that if the brand firm determines its price based on market recognition ϑ , and ϑ is influenced by JR, there is a dual effect: one impacts the downstream consumers' purchase behavior, and the other effects the upstream manufacturer's pricing behavior. No matter what effect occurs, the enterprise chooses the optimal scheme regarding the acquisition cost. Therefore, Theorem 6 further proves the robustness of the outcomes in Theorems 1-5.

Wholesale pricing: $p_{w} = k_{2}q$

In reality, the firm may determine its price based on crowdsourcing design quality q. We assume that $p_w=k_2q$. Based on the previous analysis $q=r_{_{\! m}}$, thus $p_{_{\! w}}=k_2q=k_2r_{_{\! m}}$. Then we obtain Theorem 7.

Theorem 7:

Under the mandatory sharing, the enterprise acquires MR if and only if $c_a \leq \hat{l}_2$, where $\hat{l}_2 = \left\{ \overline{r}_{\scriptscriptstyle m}^{\ 2} \left(1 + a_{\scriptscriptstyle 0} \right) \left[\left(1 + a_{\scriptscriptstyle 0} \right) \lambda^2 + 2 \left(4 a_{\scriptscriptstyle 0} - k_{\scriptscriptstyle 2} + 4 \right) \lambda + 4 \left(a_{\scriptscriptstyle 0} - 2 k_{\scriptscriptstyle 2} + 1 \right) \right] \right\} / \, 48 \xi \, ; \text{under the optional sharing, the enterprise acquires MR on crowdsourcing designs if and only if } \, c_a \leq \tilde{l}_2 \, \text{ where:}$

$$\tilde{l_{2}} = \{ \left(1 + a_{\scriptscriptstyle 0}\right) [\left(\overline{r}_{\scriptscriptstyle m} + 2r_{\scriptscriptstyle f}\right) \left(1 + a_{\scriptscriptstyle 0}\right) \left(\overline{r}_{\scriptscriptstyle m} - r_{\scriptscriptstyle f}\right)^{2} \lambda^{2} \\ + \left(1 + a_{\scriptscriptstyle 0} - k_{\scriptscriptstyle 2} + 1\right) \overline{r}_{\scriptscriptstyle m}^{\; 3} \lambda + \overline{r}_{\scriptscriptstyle m}^{\; 3} \left(a_{\scriptscriptstyle 0} - 2k_{\scriptscriptstyle 2} + 1\right) \right\} / 12 \xi \overline{r}_{\scriptscriptstyle m}$$

The enterprise's post-profit under the optional sharing is higher than the mandatory sharing, if and only if

and vice versa. The enterprise's prior profit under the mandatory sharing is higher than the optional sharing if $l > \left(\frac{1}{2} + A_2\right) \left(\tilde{l_2} - \hat{l_2}\right)$, and vice versa, where:

$$A_{2} = \frac{\left[\left(1 + a_{0}\right)\left(4r_{\!\scriptscriptstyle f}^{\; 3} + \overline{r}_{\!\scriptscriptstyle m}^{\; 3}\right)\right]\lambda^{2} + \left[2\left(4 - k_{\!\scriptscriptstyle 2} + 4a_{\!\scriptscriptstyle 0}\right)\overline{r}_{\!\scriptscriptstyle m}^{\; 3} + 2r_{\!\scriptscriptstyle f}^{\; 3}\left(1 + a_{\!\scriptscriptstyle 0} - k_{\!\scriptscriptstyle 2}\right)\right]\lambda + 4\overline{r}_{\!\scriptscriptstyle m}^{\; 3}\left(a_{\!\scriptscriptstyle 0} - 2k_{\!\scriptscriptstyle 2} + 1\right)}{3\lambda\overline{r}_{\!\scriptscriptstyle m}\left(\overline{r}_{\!\scriptscriptstyle m} - 2r_{\!\scriptscriptstyle f}\right)\!\left[\lambda\left(1 + a_{\!\scriptscriptstyle 0}\right)\!\left(\overline{r}_{\!\scriptscriptstyle m} + 2r_{\!\scriptscriptstyle f}\right) - 2k_{\!\scriptscriptstyle 2}\overline{r}_{\!\scriptscriptstyle m}\right]}$$

Theorem 7 and Figure 3 reveal that the results are consistent with the prior findings that the brand enterprise relies on acquisition costs to select its optimal scheme. MR is only one factor impacting the wholesale price; it differs from Theorem 6, wherein two factors, MR and FR, jointly impact wholesale price. Thus, the enterprise and the manufacturer may prefer the pricing policy $p_w = k_1 \vartheta$ over $p_w = k_2 q$ because the pricing policy $p_w = k_1 \vartheta$ holistically mirrors the market acceptance level of crowdsourcing designs. It implies that the firm should holistically collect various market information in a cost-saving way and share them with different agents in the crowdsourcing

process under the optional/mandatory sharing scheme. In this way, the crowdsourcing design and its finished products could be more acceptable in the market.

Consumers' Sensitivity to FR ($\lambda_{_{NR}}>\lambda_{_{non-MR}}$; $\lambda_{_{MR}}<\lambda_{_{non-MR}}$)

In practice, consumers' sensitivity to FR under non-MR acquisition ($\lambda_{\scriptscriptstyle non-MR}$) and MR acquisition ($\lambda_{\scriptscriptstyle MR}$) varies, and we assume $~\lambda_{\scriptscriptstyle MR}~/~\lambda_{\scriptscriptstyle non-MR}=\eta$. If $~1>\eta>0$, then $~\lambda_{\scriptscriptstyle MR}<\lambda_{\scriptscriptstyle non-MR}$, and $~\lambda_{\scriptscriptstyle MR}>\lambda_{\scriptscriptstyle non-MR}$ if $\eta>1$. Therefore, we have the following theorems.

Theorem 8: The enterprise's post-profit under the optional sharing is higher than the mandatory s h a r i n g , i f $\lambda > \max\{\lambda_2, \left(3p_w - r_f - a_0r_f\right) / \left[2\eta r_f\left(1 + a_0\right)\right]\}$ a n d $c_a < F_1$ o r i f $\left(3p_w - r_f - a_0r_f\right) / \left[2\eta r_f\left(1 + a_0\right)\right] < \lambda < \lambda_2$ and $c_a < \tilde{l}_3$. The enterprise's prior profit under the mandatory sharing is higher than the optional sharing, if $l > \left(\frac{1}{2} + A_4\right)\left(\tilde{l}_3 - \hat{l}_3\right)$, and vice versa. The expected consumer surplus is higher under the optional sharing than the mandatory sharing if and only if:

$$\begin{split} & l < \{A_{\!\scriptscriptstyle 5} \lambda_{_{\!non-MR}}^{2} + 4 \lambda_{_{\!non-MR}} \left(\!-3 p_{_{\!w}} \left(\!\eta \tilde{l}_{\!\scriptscriptstyle 3} - \! \left(\eta - 1\right)\!\hat{l}_{\!\scriptscriptstyle 3}\right) \overline{r}_{_{\!m}}^{2} + 6 \overline{r}_{_{\!m}} \tilde{l}_{\!\scriptscriptstyle 3} p_{_{\!w}} r_{_{\!f}} + \eta r_{_{\!f}}^{2} \tilde{l}_{\!\scriptscriptstyle 3} \left(a_{_{\!0}} r_{_{\!f}} - 3 p_{_{\!w}} + r_{_{\!f}}\right)\right) \\ & + 4 \overline{r}_{_{\!m}}^{2} (\overline{r}_{_{\!m}} \left(\!1 + \eta \lambda_{_{\!non-MR}}\right)^2 \left(\!1 + a_{_{\!0}}\right) \!- 3 p_{_{\!w}}\right) \! \left(\tilde{l}_{\!\scriptscriptstyle 3} - \hat{l}_{\!\scriptscriptstyle 3}\right) \!\} \, / \, G_{_{\!1}} \end{split}$$

and vice versa, where:

$$\begin{split} &(1+a_0)[(1+a_0)((4\eta^2-3)\overline{r_{_{m}}}^3+8r_{_{f}}^3\eta^2)\lambda_{_{non-MR}}^{}{}^2+(8\eta(1+a_0)\overline{r_{_{m}}}^3\\ &-12p_{_{w}}(\eta-1)\overline{r_{_{r}}}^2+4r_{_{f}}^2\eta(a_0r_{_{f}}-3p_{_{w}}+r_{_{f}}))\lambda_{_{non-MR}}\\ F_1 &= \frac{+4\overline{r_{_{m}}}^2(\overline{r_{_{m}}}(1+a_0)-3p_{_{w}}]}{48\xi\overline{r_{_{m}}}}\\ &\qquad \qquad \left(1+a_0\right)\!\left(\!\left(4\eta^2-3\right)\overline{r_{_{m}}}^3+4r_{_{f}}^3\eta^2\right)\!\lambda_{_{non-MR}}^{}^2\\ &\qquad \qquad +\left(8\eta\left(1+a_0\right)\overline{r_{_{m}}}^3-12p_{_{w}}\left(\eta-1\right)\overline{r_{_{m}}}^2+2r_{_{f}}^2\eta\left(a_0r_{_{f}}-3p_{_{w}}+r_{_{f}}\right)\right)\!\lambda_{_{non-MR}}\\ A_4 &= \frac{+4\overline{r_{_{m}}}^2\left(\overline{r_{_{m}}}\left(1+a_0\right)-3p_{_{w}}\right)}{3\lambda_{_{non-MR}}\overline{r_{_{m}}}\left(\overline{r_{_{m}}}-2r_{_{f}}\right)\!\left[\lambda_{_{non-MR}}\left(1+a_0\right)\!\left(\overline{r_{_{m}}}+2r_{_{f}}\right)-4p_{_{w}}\right]}\\ A_5 &= \left(8\tilde{l_3}r_{_{f}}^3\eta^2+3\hat{l_3}\overline{r_{_{m}}}^3-12\overline{r_{_{m}}}\tilde{l_3}r_{_{f}}^2\right)\!\left(1+a_0\right) \end{split}$$

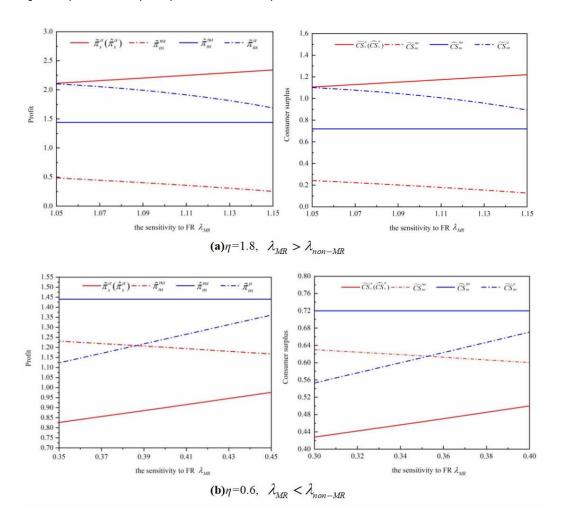
and:

$$G_{1}=3\lambda_{_{non-MR}}\overline{r}_{_{m}}\left(\overline{r}_{_{m}}-2r_{_{f}}
ight)\!\left(\lambda_{_{non-MR}}\left(\overline{r}_{_{m}}+2r_{_{f}}
ight)\!\left(1+a_{_{0}}
ight)\!-4p_{_{w}}
ight)$$

Theorem 8 shows similar results to Theorem 1-5. Meanwhile, it indicates that π plays a key role in profit and consumer surplus. To graphically demonstrate Theorem 8, we conduct a numerical study by assuming $\pi=1.8$ ($\lambda_{MR}>\lambda_{non-MR}$), or $\pi=0.6$ ($\lambda_{MR}<\lambda_{non-MR}$), $a_0=4$, $\lambda_{non-MR}=1$, $\xi=1$, $p_w=0.1$, $c_a=0.1$, $\overline{r}_m=1$, $r_m=0.3$.

Figure 4 shows that if the enterprise decides to acquire and share MR, whether the condition is π =0.6, $\lambda_{MR} < \lambda_{non-MR}$ or π =1.8, $\lambda_{MR} > \lambda_{non-MR}$, its profit $\tilde{\pi}_s^a\left(\hat{\pi}_s^a\right)$ and consumers' surplus $\widetilde{CS}_s^a\left(\widehat{CS}_s^a\right)$ increase with λ_{MR} , but the enterprise's profit $\tilde{\pi}_s^a\left(\hat{\pi}_s^a\right)$ and consumers' surplus $\widetilde{CS}_s^a\left(\widehat{CS}_s^a\right)$ are higher when $\lambda_{MR} > \lambda_{non-MR}$ (π =1.8) than $\lambda_{MR} < \lambda_{non-MR}$ (π =0.6). Conversely, if the enterprise does not acquire MR under the optional sharing, the enterprise's profit $\tilde{\pi}_{ns}^{na}$ and consumer surplus \widetilde{CS}_{ns}^{na} decrease with λ_{MR} ; meanwhile, the enterprise's profit $\tilde{\pi}_{ns}^{na}$ and consumers' surplus \widetilde{CS}_{ns}^{na} are lower when $\lambda_{MR} > \lambda_{non-MR}$ (π =1.8) than $\lambda_{MR} < \lambda_{non-MR}$ (π =0.6). Interestingly, when the enterprise does not share MR and $\lambda_{MR} < \lambda_{non-MR}$, there exists a threshold value λ_{MR}^0 (0.385), and the enterprise abandons MR acquisition below λ_{MR}^0 ; otherwise, it selects to obtain MR. This phenomenon does not occur in the setting of $\lambda_{MR} > \lambda_{non-MR}$, where the enterprise always selects to obtain MR. The reason is that

Figure 4. Impact on the enterprise's profit and consumer surplus



even though $\lambda_{MR} < \lambda_{non-MR}$, the enterprise still has a psychological threshold toward consumers' sensitivity to FR, above which the enterprise is willing to acquire MR with an aim to further prove crowdsourcing design quality.

NUMERICAL STUDIES

In this section, we study the effects of crowdsourcing design's JR r_j and the possibility of acquiring MR on the optimal choice of MR acquisition/sharing scheme.

The Impact of MR on the Joint Reviews and KOLs' Reviews

Figure 5 illustrates the impact of MR on FR and on JR under the optional and the mandatory sharing schemes. Figure 5(a) shows that under the optional sharing, there are three different areas. The first is when MR is below 0.4, the impact of MR on FR remains unchanged, and the impact on JR slightly increases with MR; this means MR plays a critical role in assessing crowdsourcing designs. The second is when the MR value falls in the interval [0.4, 0.9], MR impacts both FR and JR, but the influence on JR is more significant than that on FR. The last is when the MR value surpasses 0.9, FR and JR increase with MR, and FR values are larger than JR, which implies that the excessive values of both FR and MR do not always enhance the effect of JR on crowdsourcing designs (i.e., JR has a ceiling value).

The reason JR has a ceiling value is that it is subject to the dual effects of direct and indirect MR, owning that the firm discloses the manufacturer's MR to influence KOL's comments, thus impacting JR. Therefore, the impact of MR on JR is greater than that of FR only if $r_m < 0.9$, but if the impact of MR on JR is smaller than that of FR if $r_m > 0.9$, it indicates that the role of MR in improving JR is decreasing. In the real world, the phenomenon is that the excessively high MR could lead to potential consumers' suspicion and distrust of crowdsourcing designs, resulting in JR failure. It suggests that, the firm has to take MR or FR seriously, especially for excessive high or low MR, which should be carefully disclosed to avoid potential consumers from generating erroneous and biased perceptions. For example, in crowdsourcing innovation, Amazon first invites experts to conduct pre-evaluation and then screens out those MR with extremely high and low scores, followed by subsequent activities such as disclosing MR to KOLs.

Figure 5(b) indicates that JR and FR increase with MR under the mandatory sharing. However, when the MR value exceeds 0.55, the influence on FR is more significant than on JR; when the MR

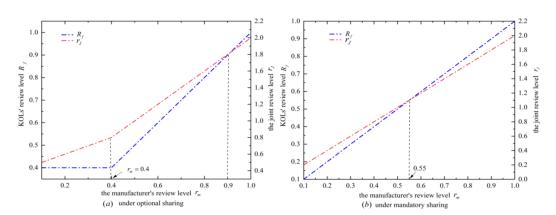


Figure 5. The impact of MR on FR and JR under optional/mandatory sharing

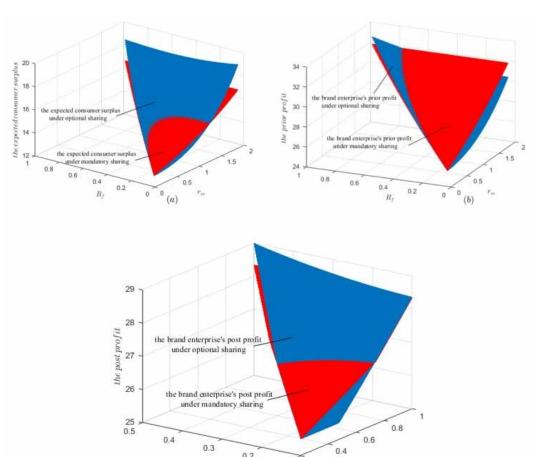
value is below 0.55, the outcome is reversed. This further verifies that excessively high MR does not always improve JR, but the mandatory sharing scheme touches the ceiling earlier than the optional sharing scheme.

The Impact of JR on Profits and Consumer Surplus

To investigate the joint effects of MR and FR on the enterprise's post and prior profits and consumer surplus, we specify $a_{_0}=10$, $\lambda=1$, $\xi=1$, $c_{_a}=2$, $p_{_w}=0.1$, l=6, and 12.

Figure 6 shows that when MR and FR are low, the enterprise's post and prior profits and consumer surplus under the mandatory sharing are higher than the optional sharing. Under this circumstance, the mandatory sharing is better for the enterprise. However, when MR and FR are high, the enterprise's post and prior profits and consumer surplus under the mandatory sharing are lower than the optional sharing. Under this circumstance, the optional sharing is better.

Figure 6 (a-c) demonstrates that the area under the mandatory sharing is the smallest one from the consumer surplus perspective (Figure 6a), followed by that from the post-profit perspective (Figure 6c), while the area in Figure 6b is the largest. Meanwhile, consumer surplus and prior profit under the mandatory sharing increase more smoothly with MR and FR than those under the optional



 R_{f}

Figure 6. The impact of JR on post/prior profit and consumer surplus

sharing. In contrast, the post profit under the mandatory/optional sharing rises sharply with MR and FR. This phenomenon offers three insights. First, from the consumer perspective, optional sharing is more favorable than mandatory sharing. Second, without considering acquisition costs, mandatory sharing is more likely to be adopted than optional sharing from the enterprise's perspective, yet such a preference diminishes with the consideration of acquisition costs. Last, optional sharing is more sensitive to MR and FR than mandatory sharing.

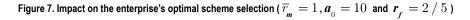
Impact on the Enterprise's Optimal Scheme Selection

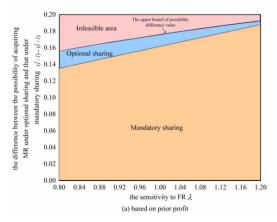
Figure 7 displays the joint impact of λ and the difference between the possibility of acquiring MR under two sharing schemes on the enterprise's selection. Specifically, Figure 7(a) illustrates this impact based on prior profit, while Figure 7(b) is based on consumer surplus. It observes that when the difference between the possibility of acquiring MR is larger, optional sharing is the better scheme with respect to the prior profit; at a medium and lower level, mandatory sharing is the optimal scheme. From the consumer perspective, when the difference is at a medium and high level, optional sharing is optimal; at a low level, mandatory sharing is the best choice. This phenomenon indicates that narrowing such a gap has a greater likelihood of choosing mandatory sharing, particularly based on consumer surplus; this move will boost the enterprise to acquire MR and then reveal it to KOLs, which beneficially generates the effect of JR on crowdsourcing designs.

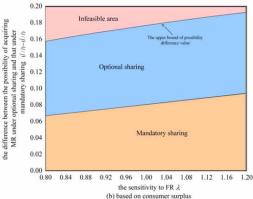
CONCLUSION

This paper considers an enterprise that crowdsources innovation designs and then outsources manufacturing to fulfill the designs. Before the crowdsourcing design is put into production, the manufacturer and KOLs may first review the design and produce comments (i.e., MR and FR). The enterprises are faced with deciding whether to let MR share with KOLs because such joint reviews would impact the purchase behaviors of prospective consumers and further influence the enterprise's production decision accordingly. Through the study, the theoretical and practical implications are summarized as follows:

(1) The results reveal that different sharing schemes dramatically influence the enterprise's optimal previous-stage MR acquisition selection, which contributes to the literature regarding how to form the joint reviews (Li et al., 2021). The optional sharing scheme enables the enterprise to







freely choose its sharing decision, which gives it more motivation to obtain MR and allows forming JR, thus benefiting the enterprise to effectively assess the crowdsourcing designs. In contrast, the mandatory sharing scheme reduces the enterprise's probability of concealing the undesirable MR, leading to more reluctance to acquire MR. As a result, it negatively impacts the generating of JR. Meanwhile, we also find that the enterprise will acquire and share MR only if its acquisition cost is relatively low and the manufacturer's comments are higher than KOLs' comments; otherwise, the MR acquisition and sharing process will stop halfway. This suggests that, with an aim to forming the effective joint reviews, the firm should first communicate with the manufacturer to improve the crowdsourcing designs to some degree before sharing with KOLs and not hastily push out the MR to KOLs.

- (2) The finding also enriches the acquiring/sharing work by examining the interaction between the manufacturer and KOLs in the crowdsourcing designs. It shows that the enterprise is inclined to select the optional sharing when the ratio of KOLs' comments to the highest manufacturer's MR is below a certain threshold value. However, once the ratio exceeds this value, meaning the KOLs' comments are closer to the manufacturer's highest comments, the enterprise will choose the mandatory rather than the optional sharing scheme. This indicates that the firm should invite each commentator, including crowdsourcees, KOLs, and the manufacturer, to freely exchange improvement suggestions in the crowdsourcing process, thus narrowing their divergence in comments and facilitating the production of JR to improve crowdsourcing quality.
- (3) We uncover that JR and FR increase with MR. However, when the MR value exceeds a certain threshold, the influence on FR is more significant than on JR; when the MR value is below the threshold, the outcome is reversed, which implies that JR has a ceiling value. The excessively high KOLs' reviews (FR) and MR do not always enhance the effect of JR on crowdsourcing designs; meanwhile, mandatory sharing touches the ceiling earlier than optional sharing. This suggests that, although joint reviews can effectively evaluate the quality of crowdsourcing design, they should not be abused or overused, otherwise they will be counterproductive.
- (4) The results also reveal that the enterprise prefers the market-recognition over the design-quality pricing policy because market recognition involves related factors, including crowdsourcing-design quality. This implies that the firm should holistically collect various market information and share it with different agents in the crowdsourcing process under e optional/mandatory sharing. In this way, the crowdsourcing designs and their finished products could be more acceptable in the market.

There are some limitations in this paper that offer opportunities for future research. First, the desirable crowdsourcing design quality needs more time and multiple rounds of iteration to improve. How to address these issues is an interesting and worthy future research direction. Second, the manufacturer only acts as a MR information reviewer and not as a decision-maker in this paper. Future studies could take this into account. Finally, the reviews considered in this study are only two-dimensional; multi-dimensional reviews, including peer reviews can be investigated in future research to evaluate their impact on firms' decisions.

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APPENDIX

Proof of Lemma 1

Solving $\tilde{\pi}^a_s > \tilde{\pi}^a_{ns}$, then obtain $r_m > r_{f}$.

Proof of Theorem 1:

$$=\frac{\left(1+a_{_{0}}\right)\left[\left(\overset{-}{r_{_{m}}}+2r_{_{f}}\right)\left(1+a_{_{0}}\right)\left(\overset{-}{r_{_{m}}}-r_{_{f}}\right)^{2}\lambda^{2}+\left(\overset{-}{2a_{_{0}}+2}\overset{-}{r_{_{m}}}r_{_{w}}+6r_{_{m}}r_{_{f}}p_{_{w}}}{-3r_{_{m}}p_{_{w}}+r_{_{f}}^{2}\left(\left(1+a_{_{0}}\right)r_{_{f}}-3p_{_{w}}\right)\right)}\lambda+\overset{-}{r_{_{m}}}^{2}\left(\overset{-}{r_{_{m}}}\left(1+a_{_{0}}\right)-3p_{_{w}}\right)}{12\mathcal{E}r_{_{m}}}-c_{_{0}}$$

Solving $\tilde{\pi}_{a} \geq \tilde{\pi}_{na}$, and obtaining:

$$c_{a} \leq \frac{\left(1+a_{_{0}}\right)\left[\left(\overset{-}{r}_{m}+2r_{_{f}}\right)\left(1+a_{_{0}}\right)\left[\overset{-}{r}_{m}-r_{_{f}}\right)^{2}\lambda^{2}+\left(\frac{\left(2a_{_{0}}+2\right)\overset{-}{r}_{m}^{3}}{-3r_{_{m}}p_{_{w}}+6r_{_{m}}r_{_{f}}p_{_{w}}}{+r_{_{f}}^{2}\left(\left(1+a_{_{0}}\right)r_{_{f}}-3p_{_{w}}\right)}\right)\lambda+\overset{-}{r_{_{m}}}^{2}\left(\overset{-}{r}_{m}\left(1+a_{_{0}}\right)-3p_{_{w}}\right)}{12\xi r_{_{m}}}$$

Proof of Theorem 2:

$$\hat{\pi}_{a} - \hat{\pi}_{na} = \frac{\bar{r}_{m} \left(1 + a_{0}\right) \left[\bar{r}_{m} \left(1 + a_{0}\right) \left(\lambda^{2} + 8\lambda + 4\right) - 12p_{w}\right]}{48\xi} - c_{a}$$

$$\text{Solving } \hat{\pi_a} \geq \hat{\pi}_{\scriptscriptstyle na} \text{ and obtaining } c_a \leq \frac{\bar{r}_{\scriptscriptstyle m} \left(1 + a_{\scriptscriptstyle 0}\right) \left[\bar{r}_{\scriptscriptstyle m} \left(1 + a_{\scriptscriptstyle 0}\right) \left(\lambda^2 + 8\lambda + 4\right) - 12 p_{\scriptscriptstyle w}\right]}{48 \xi}$$

Proof of Lemma 2 is shown within this paper

Proof of Lemma 3:

$$P\left(c_{_{a}}\leq\tilde{l}\right)\cdot P(r_{_{m}}>r_{_{f}})>P(c_{_{a}}\leq\hat{l})\ \ \text{can be reduced to}\ \ r_{_{f}}\ /\ \bar{r}_{_{m}}<1-\hat{l}\ /\ \tilde{l}\ \ .$$
 Based on above analysis, Lemma 2 holds.

Proof of Theorem 3:

(a) When
$$\lambda > \lambda_1$$
 (i.e., $\tilde{l} > \hat{l}$):

 $\text{When the acquisition cost } c_{\scriptscriptstyle a} \in \left(\tilde{l}\,, l\right], \, \hat{\pi} - \tilde{\pi} = \frac{\lambda \left(1 + a_{\scriptscriptstyle 0}\right) \left(\bar{r}_{\scriptscriptstyle m} - 2r_{\scriptscriptstyle f}\right) \left[\lambda \left(1 + a_{\scriptscriptstyle 0}\right) \left(\bar{r}_{\scriptscriptstyle m} + 2r_{\scriptscriptstyle f}\right) - 4p_{\scriptscriptstyle w}\right]}{16\xi} > 0$

due to $r_{_f} < \overline{r_{_m}} / 2$ and $4p_{_w} < \lambda \left(1 + a_{_0}\right) \left(\overline{r_{_m}} + 2r_{_f}\right)$ because the wholesale price is lower than the retail price. Thus, under this circumstance, $\hat{\pi} > \tilde{\pi}$ when $c_{_a} > \tilde{l}$.

When the acquisition cost $c_a \in (\hat{l}, \tilde{l}]$,

$$\begin{split} &\left(1+a_{_{0}}\right)\!\left[\!\left(\!\left(1+a_{_{0}}\right)\!\stackrel{-}{r_{_{m}}}^{3}+8r_{_{f}}^{3}\left(1+a_{_{0}}\right)\!\right)\!\lambda^{2}+\left(\!\left(8a_{_{0}}+8\right)\!\stackrel{-}{r_{_{m}}}^{3}\right.\right.\\ &\left.\tilde{\pi}-\hat{\pi}=\frac{+4r_{_{f}}^{2}\left(a_{_{0}}r_{_{f}}+r_{_{f}}-3p_{_{w}}\right)\!\lambda+4\left(\!\left(1+a_{_{0}}\right)\!\stackrel{-}{r_{_{m}}}-3p_{_{w}}\right)\!\stackrel{-}{r_{_{m}}}^{2}\right]}{48\xi r_{_{m}}}-c_{_{a}}=F-c_{_{a}} \end{split}$$

where:

$$\begin{split} &\left(1+a_{_{0}}\right)\!\!\left[\!\left(\!\left(1+a_{_{0}}\right)\!\!\stackrel{-}{r_{_{m}}}^{3}+8r_{_{f}}^{\,3}\left(1+a_{_{0}}\right)\!\right]\!\lambda^{2}+\left(\!\left(8a_{_{0}}+8\right)\!\!\stackrel{-}{r_{_{m}}}^{3}\right.\\ &\left.F=\frac{+4r_{_{f}}^{\,2}\left(a_{_{0}}r_{_{f}}+r_{_{f}}-3p_{_{w}}\right)\!\lambda+4\left(\!\left(1+a_{_{0}}\right)\!\!\stackrel{-}{r_{_{m}}}-3p_{_{w}}\right)\!\!\stackrel{-}{r_{_{m}}}^{2}\right]}{48\xi r_{_{m}}} \end{split}$$

We can easily know that:

$$\tilde{l}-F = \frac{\left[\left(\bar{r}_{\scriptscriptstyle m} + 2r_{\scriptscriptstyle f}\right)\left(1 + a_{\scriptscriptstyle 0}\right)\lambda - 4p_{\scriptscriptstyle w}\right]\lambda\left(\bar{r}_{\scriptscriptstyle m} - 2r_{\scriptscriptstyle f}\right)\left(1 + a_{\scriptscriptstyle 0}\right)}{16\xi} > 0$$

due to
$$\, r_{\!\scriptscriptstyle f} < \frac{\bar{r}_{\scriptscriptstyle m}}{2} \,$$
 and $\, (\overline{r}_{\!\scriptscriptstyle m} + 2 r_{\!\scriptscriptstyle f}) (1 + a_{\!\scriptscriptstyle 0}) \lambda > 4 \, p_{\scriptscriptstyle w} \,.$

$$\text{If} \quad F \leq \hat{l} \quad \text{(i.e.,} \quad \lambda < \frac{3p_w - r_f - a_0 r_f}{2r_f \left(1 + a_0\right)} \; \text{)}, \quad \tilde{\pi} - \hat{\pi} = F - c_a < F - \hat{l} \leq 0 \; . \quad \text{If} \quad F > \hat{l} \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{If} \quad F > \hat{l} = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f + c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f - c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f - c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f - c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f - c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f - c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f - c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f - c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f - c_a < f - c_a < f - \hat{l} \leq 0 \; . \quad \text{(i.e.,} \quad f = f - c_a < f - c_a < f$$

$$\lambda > \frac{3p_{_w} - r_{_f} - a_{_0}r_{_f}}{2r_{_f}\left(1 + a_{_0}\right)}$$
), solving $\,\tilde{\pi} > \hat{\pi}$, we obtain $\,\hat{l} < c_{_a} < F$.

Thus, under this circumstance, $\ \tilde{\pi} > \hat{\pi} \ \ \text{if} \ \ \hat{l} < c_{_a} < F \ \ \text{and} \ \ \lambda > \frac{3p_{_w} - r_{_f} - a_{_0}r_{_f}}{2r_{_c}(1+a_{_0})} \ .$

$$\text{When the acquisition cost } \ c_{_{a}} \in \left[0, \hat{l}\,\right], \ \tilde{\pi} - \hat{\pi} = \frac{\lambda r_{_{f}}^{\,\,2} \left(1 + a_{_{0}}\right) \left[r_{_{f}} \left(1 + 2\lambda\right) \left(1 + a_{_{0}}\right) - 3p_{_{w}}\right]}{12 r_{_{m}} \mathcal{E}} \,.$$

Solving
$$\, \tilde{\pi} > \hat{\pi}$$
 , we obtain $\, \lambda > \dfrac{3p_{_w} - r_{_f} - a_{_0}r_{_f}}{2r_{_f}\left(1 + a_{_0}\right)} \, .$

Thus, under this circumstance,
$$\, \tilde{\pi} > \hat{\pi} \,$$
 if $\, \lambda > \dfrac{3 \, p_{_{w}} - r_{_{\! f}} - a_{_{\! 0}} r_{_{\! f}}}{2 r_{_{\! f}} \left(1 + a_{_{\! 0}}\right)} \,$ and $\, c_{_{\! a}} < \hat{l} \,$.

Therefore, from the above analysis, we can easily know that $\tilde{\pi} > \hat{\pi}$ if $\lambda > \max\{\lambda_{_{\! 1}}, \frac{3p_{_{\! w}} - r_{_{\! f}} - a_{_{\! 0}}r_{_{\! f}}}{2r_{_{\! f}}\left(1 + a_{_{\! 0}}\right)}\}$ and $c_{_a} < F$.

(b) When $\lambda < \lambda_1$ (i.e., $\tilde{l} < \hat{l}$):

When the acquisition cost $c_a \in (\hat{l}, l]$, $\hat{\pi} - \tilde{\pi} = \frac{\lambda \left(1 + a_0\right) \left(\bar{r}_m - 2r_f\right) \left[\lambda \left(1 + a_0\right) \left(\bar{r}_m + 2r_f\right) - 4p_w\right]}{16\xi} > 0$ due to $r_f < \bar{r}_m \ / \ 2$ and $4p_w < \lambda \left(1 + a_0\right) \left(\bar{r}_m + 2r_f\right)$ because the wholesale price is lower than the retail price.

Thus, under this circumstance, $\hat{\pi} > \tilde{\pi}$ when $c_a > \hat{l}$, when the acquisition cost $c_a \in \left(\tilde{l},\hat{l}\right]$:

$$\begin{split} \hat{\pi} - \tilde{\pi} &= \frac{(1 + a_0)[(\overline{r_{\scriptscriptstyle m}}^2 - 3r_{\scriptscriptstyle f}^2)(1 + a_0)\lambda^2 + (2(1 + a_0)\overline{r_{\scriptscriptstyle m}}^2 - 3\overline{r_{\scriptscriptstyle m}}p_{\scriptscriptstyle w} + 6p_{\scriptscriptstyle w}r_{\scriptscriptstyle f})\lambda + \overline{r_{\scriptscriptstyle m}}((1 + a_0)\overline{r_{\scriptscriptstyle m}} - 3p_{\scriptscriptstyle w})]}{12\xi} - c_{\scriptscriptstyle a} \\ &= H - c_{\scriptscriptstyle a} \geq H - \hat{l} = \frac{\lambda(1 + a_0)(\overline{r_{\scriptscriptstyle m}} - 2r_{\scriptscriptstyle f})[\lambda(1 + a_0)(\overline{r_{\scriptscriptstyle m}} + 2r_{\scriptscriptstyle f}) - 4p_{\scriptscriptstyle w}]}{16\xi} > 0 \end{split}$$

due to $\left.\lambda\left(1+a_{_{0}}\right)\left(\stackrel{-}{r}_{^{m}}+2r_{_{\!f}}\right)>4p_{_{w}}$, where:

$$H = \frac{\left(1 + a_{\scriptscriptstyle 0}\right)\!\left[\!\left(\bar{r}_{\scriptscriptstyle m}^{-2} - 3r_{\scriptscriptstyle f}^{\,2}\right)\!\left(1 + a_{\scriptscriptstyle 0}\right)\lambda^{2} + \left(2\left(1 + a_{\scriptscriptstyle 0}\right)\bar{r}_{\scriptscriptstyle m}^{-2} - 3\bar{r}_{\scriptscriptstyle m}p_{\scriptscriptstyle w} + 6p_{\scriptscriptstyle w}r_{\scriptscriptstyle f}\right)\lambda + \bar{r}_{\scriptscriptstyle m}\left(\left(1 + a_{\scriptscriptstyle 0}\right)\bar{r}_{\scriptscriptstyle m} - 3p_{\scriptscriptstyle w}\right)\right]}{12\xi}$$

thus, under this circumstance, $\,\hat{\pi} > \tilde{\pi} \,\,$ if $\,\tilde{l} < c_{_{a}} < \hat{l} \,\,$.

When the acquisition cost
$$c_a \in \left[0, \tilde{l}\right], \ \tilde{\pi} - \hat{\pi} = \frac{\lambda r_{_f}^{\,\, 2} \left(1 + a_{_0}\right) \left[r_{_f} \left(1 + 2\lambda\right) \left(1 + a_{_0}\right) - 3p_{_w}\right]}{12 r_{_m} \xi}$$

Solving $\tilde{\pi} > \hat{\pi}$ obtain $\lambda > \frac{3p_w - r_f - a_0 r_f}{2r_f \left(1 + a_0\right)}$. Thus, under this circumstance, $\tilde{\pi} > \hat{\pi}$ if $\lambda > \frac{3p_w - r_f - a_0 r_f}{2r_f \left(1 + a_0\right)}$ and $c_a < \tilde{l}$.

Therefore, from the above analysis, we can easily know that $\tilde{\pi} > \hat{\pi}$ if $\frac{3p_w - r_f - a_0 r_f}{2r_f \left(1 + a_0\right)} < \lambda < \lambda_1$ and $c_a < \tilde{l}$. Based on above analysis, Theorem 3 holds.

Proof of Theorem 4:

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$$\text{where } N = \frac{\overset{-}{r_m\lambda}\left(4p_w - \lambda\left(1+a_0\right)\left(\overset{-}{r_m}+2r_f\right)\right)\left(1+a_0\right)\left(\overset{-}{r_m}-2r_f\right)}{16\xi} < 0 \quad \text{due to} \quad r_f < \overset{-}{r_m} \ / \ 2 \quad \text{and} \quad p_w < \left(1+a_0\right)\lambda r_f \, .$$

Thus, we can easily know that:

$$E\left[\hat{\pi}\right] - E\left[\tilde{\pi}\right] > 0 \text{ if } l > \left(\frac{1}{2} + \frac{\left[\left(1 + a_{_{0}}\right)\left(4r_{_{f}}^{3} + \overline{r_{_{m}}}^{3}\right)\right]\lambda^{2} + \left[8\left(1 + a_{_{0}}\right)\overline{r_{_{m}}}^{3}}{3\lambda\overline{r_{_{m}}}\left(\overline{r_{_{m}}} - 3p_{_{w}} + r_{_{f}}\right)\right]\lambda + 4\overline{r_{_{m}}}^{2}\left[\left(1 + a_{_{0}}\right)\overline{r_{_{m}}} - 3p_{_{w}}\right]}{\left[\lambda\left(1 + a_{_{0}}\right)\left(\overline{r_{_{m}}} + 2r_{_{f}}\right) - 4p_{_{w}}\right]}\right]\left(\tilde{l} - \hat{l}\right)$$

Based on above analysis, Theorem 4 holds.

Proof of Theorem 5:

(1) The enterprise acquires MR ($s_{\scriptscriptstyle A}=a$); in the last stage, we can easily derive that the enterprise's optimal price that maximizes its profit $\pi=\left(p-p_{\scriptscriptstyle w}\right)Q-c_{\scriptscriptstyle a}$, where $Q=\left(\vartheta-p\right)/\xi$, is $p^*=\left(\vartheta+p_{\scriptscriptstyle w}\right)/2$ from the first-order condition $\partial\pi/\partial p=0$.

Thus, the enterprise's optimal product quantities are:

$$\boldsymbol{Q}^{*}=\left(\vartheta-\boldsymbol{p}^{*}\right)/\left.\boldsymbol{\xi}=\left\{\vartheta-\left[\left(\vartheta+\boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\right]\right\}/\left.\boldsymbol{\xi}=\left(\vartheta-\boldsymbol{p}_{_{\boldsymbol{w}}}\right)/\left.2\boldsymbol{\xi}\right\}\right\}$$

We know that the consumer's quadratic utility function is $U=\vartheta Q-\xi Q^2/2-pQ$. Thus, under this circumstance, the consumer surplus can be expressed as:

$$\begin{split} U &= \vartheta \cdot \left[\left(\vartheta - p_{_{\boldsymbol{w}}} \right) / \, 2\xi \right] - \left(\xi \, / \, 2 \right) \cdot \left[\left(\vartheta - p_{_{\boldsymbol{w}}} \right) / \, 2\xi \right]^2 \, - \\ &\left[\left(\vartheta + p_{_{\boldsymbol{w}}} \right) / \, 2 \right] \cdot \left[\left(\vartheta - p_{_{\boldsymbol{w}}} \right) / \, 2\xi \right] = \left(\vartheta - p_{_{\boldsymbol{w}}} \right)^2 \, / \, 8\xi \end{split}$$

For the optional sharing, from the previous analysis, we know that if the enterprise chooses to share the MR information to KOLs in the second stage, $\vartheta = \tilde{\vartheta}^a_s = (1 + a_0)(r_m + \lambda r_m)$.

Under this circumstance, the consumer surplus is $\widetilde{CS}_s^a = [(1+a_0)(r_m+\lambda r_m)-p_w]^2/8\xi$. Otherwise, $\vartheta=\tilde{\vartheta}_{ns}^a=\left(1+a_0\right)\left(r_m+\lambda r_f\right)$ and the consumer surplus is $\widetilde{CS}_{ns}^a=[\left(1+a_0\right)\left(r_m+\lambda r_f\right)-p_w]^2/8\xi$. For the mandatory sharing, from the previous analysis, we know that $\vartheta=\hat{\vartheta}_s^a=\left(1+a_0\right)\left(r_m+\lambda r_m\right)$ under this circumstance. Thus, the consumer surplus is $\widehat{CS}_s^a=[\left(1+a_0\right)\left(r_m+\lambda r_m\right)-p_w]^2/8\xi$.

(2) The enterprise does not acquire MR ($s_{_A}=na$). In the last stage, we can easily derive that the enterprise's optimal price that maximizes its profit $\pi=\left(p-p_{_w}\right)Q$, where $Q=\left(\vartheta-p\right)/\xi$, is $p^*=\left(\vartheta+p_{_w}\right)/2$ from the first-order condition $\partial\pi/\partial p=0$.

Thus, the enterprise's optimal product quantities are:

$$\boldsymbol{Q}^{*} = \left(\vartheta - \boldsymbol{p}^{*}\right) / \, \boldsymbol{\xi} = \left\{\vartheta - \left[\left(\vartheta + \boldsymbol{p}_{\scriptscriptstyle \boldsymbol{w}}\right) / \, 2\right]\right\} / \, \boldsymbol{\xi} = \left(\vartheta - \boldsymbol{p}_{\scriptscriptstyle \boldsymbol{w}}\right) / \, 2\boldsymbol{\xi}$$

We know that the consumer's quadratic utility function is $U = \vartheta Q - \xi Q^2 / 2 - pQ$. Thus, under this circumstance, the consumer surplus can be expressed as:

$$\begin{split} U &= \vartheta \cdot \left[\left(\vartheta - p_{_{\boldsymbol{w}}} \right) / \, 2\xi \, \right] - \left(\xi \, / \, 2 \right) \cdot \left[\left(\vartheta - p_{_{\boldsymbol{w}}} \right) / \, 2\xi \, \right]^2 - \left[\left(\vartheta + p_{_{\boldsymbol{w}}} \right) / \, 2 \right] \cdot \left[\left(\vartheta - p_{_{\boldsymbol{w}}} \right) / \, 2\xi \right) \right] = \left(\vartheta - p_{_{\boldsymbol{w}}} \right)^2 \, / \, 8\xi \end{split}$$

For the optional sharing, from the previous analysis in this paper, we know that $\vartheta = \widetilde{\vartheta}_{ns}^{na} = \lambda \left(1 + a_0\right) r_f$ under this circumstance. Thus, the consumer surplus is $\widetilde{CS}_{ns}^{na} = \left[\lambda \left(1 + a_0\right) r_f - p_w\right]^2 / 8\xi$. For the mandatory sharing, from the previous analysis, we know that $\vartheta = \widehat{\vartheta}_{ns}^{na} = \lambda (1 + a_0) (\overline{r}_m / 2)$ under this circumstance. Thus, the consumer surplus is $\widehat{CS}_{ns}^{na} = \left[\lambda \left(1 + a_0\right) \left(\overline{r}_m / 2\right) - p_w\right]^2 / 8\xi$.

Finally, by comparing the expected consumer surplus under mandatory sharing and optional sharing, we can easily derive:

$$l < \{A\lambda^2 + 4\lambda \tilde{l} \left(\left(1 + a_0 \right) r_{_{\! f}}^{^{\; 3}} - 3 p_{_{\! w}} \left(\begin{matrix} \bar{r}_m - r_{_{\! f}} \end{matrix} \right)^2 \right) + 4 \bar{r}_m^{^{\; - 2}} \left(\begin{matrix} \bar{r}_m \left(1 + 2\lambda \right) \left(1 + a_0 \right) - 3 p_{_{\! w}} \right) \left(\tilde{l} - \hat{l} \right) \} \, / \, B \, . \,$$

where:

$$A = (4 { \begin{pmatrix} \overset{-}{r}_{\scriptscriptstyle m} + 2r_{\scriptscriptstyle f} \end{pmatrix} } { \begin{pmatrix} \overset{-}{r}_{\scriptscriptstyle m} - r_{\scriptscriptstyle f} \end{pmatrix} }^2 \tilde{l} - { \begin{matrix} \overset{-}{r}_{\scriptscriptstyle m} ^3 \hat{l} \\ \end{matrix} } { \begin{pmatrix} 1 + a_{\scriptscriptstyle 0} \end{pmatrix} }$$

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and $B=3\lambda \overset{-}{r_m} (\overset{-}{r_m}-2r_{_f}) \Big(\lambda (\overset{-}{r_m}+2r_{_f}) \big(1+a_{_0}\big)-4p_{_w}\Big)$, from the inequality $\hat{u}_{_{cs}}<\tilde{u}_{_{cs}}$ after some algebra simplifications.

From the above analysis, we obtain Theorem 5.

Proof of Theorem 6-8

The proving process of Theorem 6-8 is similar to Theorems 1-5.